Uncertainty in Bipolar Preference Problems

Maria Silvia Pini, Francesca Rossi and Brent Venable

Department of Pure and Applied Mathematics, University of Padova, Italy
E-mail: {mpini,frossi,kvenable}@math.unipd.it

Abstract Real-life problems can present several kinds of preferences, and may also contain some uncertain parts. In this paper we focus on problems with both positive and negative totally ordered preferences, and with some uncontrollable variables, which model the uncertainty of the problem. We call such problems *uncertain bipolar problems* (UBPs). After defining such problems, we propose to handle them by extending existing techniques to handle bipolar problems (BPs) and problems with uncertainty. In particular, we first eliminate the uncertainty of the problem, transforming a UBP into a BP. Then we solve the BP by associating to each solution both a degree of preference and a degree of robustness. Suitable semantics are then defined to order the solutions according to different attitudes with respect to these two notions.

1 Introduction

Real-life problems present several kinds of preferences. In this paper we focus on problem which may present both negative and positive preferences [3]. Thus, each partial instantiation within a constraint will be associated to either a positive or a negative preference.

For example, when buying a house, we may like very much to live in the country, but we may also don't like to have to take a bus to go to work, and be indifferent to the color of the house. Thus we will give a preference level (either positive, or negative, or indifference) to each feature of the house, and then we will look for a house which overall has the best combined preference.

Moreover, many real-life situations contain some form of uncertainty. In this paper we model uncertainty by the presence of so-called *uncontrollable* variables. This means that the value of such variables will not be decided by us, but by Nature. A typical example, in the context of satellite scheduling or weather prediction, is a variable representing the time when clouds will disappear. Although we cannot choose the value for such variables, usually we have some information on the plausibility of the different values. This is modelled in this paper by a possibility distribution over the domains of such variables.

In this paper we focus on problems with this kind of uncertainty, that present both positive and negative preferences. We call them *uncertain bipolar problems*.

We tackle such problems by adapting and extending existing techniques to handle bipolar problems [3] and problems with preferences and uncertainty [8, 12].

When we have only negative preferences, uncertainty can be eliminated by transforming constraints among controllable and uncontrollable variables into suitable constraints on controllable variables only [12]. When we consider also positive preferences, a similar technique can be used, while maintaining similar properties, despite the fact that positive and negative preferences are combined by different operators.

The resulting problem is then a bipolar problem (BP) where, however, each partail instantiation can have both a positive and a negative preference. Such a pair of elements is then used to associate to each solution an overall preference level and an overall robustness level.

Compensation of positive with negative preferences can be done via an operator which is not associative. This does not allow for preference compensation within the constraints. However, preference compensation can be perfomed at the level of complete solutions, thus allowing us to associate two elements to each solution: a preference degree and a robustness degree. Depending on the attitude we have towards risk, we can then order solutions by using a Pareto or a lexicographic approach over such two degrees.

2 Background

In this section we give an overview of the background on which our work is based. First, we present a formalism for representing soft preferences, i.e., the semiring-based soft constraints [2]. Then, we describe the formalism for modelling bipolar preferences [3]. Finally, we present a formalism for representing uncertain preference problems [12].

2.1 Soft constraints

A soft constraint [2] is a classical constraint [6] where each instantiation of its variables has an associated value from a (totally or partially ordered) set. This set has two operations, which makes it similar to a semiring, and is called a c-semiring. A c-semiring is a tuple $(A, +, \times, \mathbf{0}, \mathbf{1})$ where: A is a set and $\mathbf{0}, \mathbf{1} \in A$; $+$ is commutative, associative, idempotent, $\mathbf{0}$ is its unit element, and $\mathbf{1}$ is its absorbing element; $\times$ is associative, commutative, distributes over $+$, $\mathbf{1}$ is its unit element and $\mathbf{0}$ is its absorbing element. Consider the relation $\leq_S$ over A such that $a \leq_S b$ iff $a + b = b$. Then: $\leq_S$ is a partial order; $+$ and $\times$ are monotone on $\leq_S$; $\mathbf{0}$ is its minimum and $\mathbf{1}$ its maximum; $(A, \leq_S)$ is a lattice and, $\forall\, a, b \in A$, $a + b = lub(a, b)$. Moreover, if $\times$ is idempotent, then $(A, \leq_S)$ is a distributive lattice and $\times$ is its glb. Informally, the relation $\leq_S$ gives us a way to compare (some of the) tuples of values and constraints. In fact, when we have $a \leq_S b$, we will say that *b is better than a*.

Given a c-semiring $S = (A, +, \times, \mathbf{0}, \mathbf{1})$, a fi nite set D (the domain of the variables), and an ordered set of variables V, a constraint is a pair $\langle def, con \rangle$ where $con \subseteq V$ and $def : D^{|con|} \rightarrow A$. Therefore, a constraint specifi es a set of variables (the ones in con), and assigns to each tuple of values of D of these variables an element of A. A soft constraint satisfaction problem (SCSP) is just a set of soft constraints over a set of variables. For example, fuzzy CSPs [10] and weighted CSPS [2] are SCSPs that can be modeled by choosing resp. c-semirings $S_{FCSP} = ([0, 1], max, min, 0, 1)$ and $S_{WCSP} = (\Re^+, min, sum, +\infty, 0)$.

2.2 Bipolar preference problems

We present a formalism for handling with positive and negative preferences [3].

Negative preferences. The structure, which is used to model negative preferences is exactly a c-semiring [2] as described in the previous section. In fact, in a c-semiring there is an element which acts as indifference, that is $\mathbf{1}$, since $\forall a \in A, a \times \mathbf{1} = a$, and the combination between negative preferences goes down in the ordering (in fact, $a \times b \leq a, b$), that is a desired property. This interpretation is very natural when considering, for example, the weighted c-semiring $(R^+, min, +, +\infty, 0)$. In fact, in this case the real numbers are costs and thus negative preferences. The sum of different costs is worse in general w.r.t. the ordering induced by the additive operator (that is, min) of the c-semiring. From now on, a standard c-semiring will be used to model negative preferences, denoted as: $(N, +_n, \times_n, \perp_n, \top_n)$.

Positive preferences. When dealing with positive preferences, two main properties should hold: combination should bring to better preferences, and indifference should be lower than all the other positive preferences. These properties can be found in the following structure. A positive preference structure is a tuple $(P, +_p, \times_p, \perp_p, \top_p)$ s. t. P is a set and $\top_p$, $\perp_p \in P$; $+_p$, the additive operator, is commutative, associative, idempotent, with $\perp_p$ as its unit element ($\forall a \in P$, $a +_p \perp_p = a$) and $\top_p$ as its absorbing element ($\forall a \in P$, $a +_p \top_p = \top_p$); $\times_p$, the multiplicative operator, is associative, commutative and distributes over $+_p$ ($a \times_p (b +_p c) = (a \times_p b) +_p (a \times_p c)$), with $\perp_p$ as its unit element and $\top_p$ as its absorbing element[1].

The additive operator of this structure has the same properties as the corresponding one in c-semirings, and thus it induces a partial order over P in the usual way: $a \leq_p b$ iff $a +_p b = b$. This allows to prove that $+_p$ is monotone ($\forall a, b, d \in P$ s. t. $a \leq_p b$, $a \times_p d \leq_p b \times_p d$) and that it is the least upper bound in the lattice $(P, \leq_p)$ ($\forall a, b \in P$, $a \times_p b \geq_p a +_p b \geq_p a, b$).

On the other hand, $\times_p$ has different properties w.r.t. $\times_n$: the best element in the ordering ($\top_p$) is now its absorbing element, while the worst element ($\perp_p$) is its unit element. $\perp_p$ models indifference. These are exactly the desired properties for the combination and for indifference w.r.t. positive preferences. An example of a positive preference structure is ($\Re^+$,max,sum,0,$+\infty$), where preferences are positive real numbers aggregated with sum and compared with max.

Bipolar preference structure. For handling both positive and negative preferences in [3] has been defi ned a structure, which is called *bipolar preference structure*, that combines the two structures described in sections 2.2 and 2.2. A bipolar preference structure is a tuple $(N, P, +, \times, \perp, \Box, \top)$ where, $(P, +_{|P}, \times_{|P}, \Box, \top)$ is a positive preference structure; $(N, +_{|N}, \times_{|N}, \perp, \Box)$ is a c-semiring; $+ : (N \cup P)^2 \longrightarrow (N \cup P)$ is an operator s. t. $a_n + a_p = a_p, \forall a_n \in N$ and $a_p \in P$; it induces a partial ordering on $N \cup P$: $\forall a, b \in P \cup N$, $a \leq b$ iff $a + b = b$; $\times : (N \cup P)^2 \longrightarrow (N \cup P)$ (called the

[1] In fact, the absorbing nature of $\top_p$ can be derived from the other properties.

compensation operator) is a commutative and monotone ($\forall a, b, c \in N \cup P$, if $a \leq b$, then $a \times c \leq b \times c$) operator.

Bipolar preference structures generalize both c-semirings and positive structures. In fact, when $\Box = \top$, we have a c-semiring and, when $\Box = \bot$, we have a positive structure. Given the way the ordering is induced by $+$ on $N \cup P$, easily, we have $\bot \leq \Box \leq \top$. Thus, there is a unique maximum element (that is, $\top$), a unique minimum element (that is, $\bot$); the element $\Box$ is smaller than any positive preference and greater than any negative preference, and it is used to model indifference.

A bipolar preference structure allows to have a richer structure for one kind of preference, that is common in real-life problems. In fact, we can have different lattices $(P, \leq_p)$ and $(N, \leq_n)$. In the following, we will write $+_n$ instead of $+_{|N}$ and $+_p$ instead of $+_{|P}$. Similarly for $\times_n$ and $\times_p$. When $\times$ is applied to a pair in $(N \times P)$, we will sometimes write $\times_{np}$ and we will call it compensation operator.

From the monotonicity of the combination operator follows that the combination of a positive and a negative preference is a preference which is higher than, or equal to, the negative one and lower than, or equal to, the positive one. Possible choices for combining strictly positive with strictly negative preferences are thus the average, the median, the min or the max operator. Moreover, by monotonicity, if $\top \times \bot = \bot$, then $\forall p \in P, p \times \bot = \bot$. Similarly, if $\top \times \bot = \top$, then $\forall n \in N, n \times \top = \top$.

In general, operator $\times$ may be not associative. For example, if the result of $\top \times \bot$ is different from $\top$ or $\bot$, or if there are $p \in P - \{\top, \Box\}, n \in N - \{\bot, \Box\}$ s.t. $p \times n = \Box$ and $\times_n$ or $\times_p$ is idempotent, then $\times$ is not associative. Since these conditions often occur in practice, it is not reasonable to require associativity of $\times$.

An example of bipolar structure is the tuple (N=$[-1, 0]$, P=$[0, 1]$, $+$=max, $\times$, $\bot$=-1, $\Box$=0, $\top$=1), where $\times$ is such that $\times_p$= max, $\times_n$=min and $\times_{np}$=sum. Negative preferences are between -1 and 0, positive preferences between 0 and 1, compensation is sum, and the order is given by max. In this case $\times$ is not associative.

Bipolar preference problems. A bipolar constraint is just a constraint where each assignment of values to its variables is associated to one of the elements in a bipolar preference structure. Given a bipolar preference structure $(N, P, +, \times, \bot, \Box, \top)$ a finite set D (the domain of the variables), and an ordered set of variables V, a constraint is a pair $\langle def, con \rangle$ where $con \subseteq V$ and $def : D^{|con|} \rightarrow (N \cup P)$. A bipolar CSP (V, C) is then just a set of variables V and a set of bipolar constraints C over V. A solution of a bipolar CSP (V, C) is a complete assignment to all variables in V, say s, with an associated preference $pref(s) = (p_1 \times_p \ldots \times_p p_k) \times (n_1 \times_n \ldots \times_n n_l)$, where, for $i := 1, \ldots, k$ $p_i \in P$, for $j := 1, \ldots, l$ $n_j \in N$, $\exists \langle def_i, con_i \rangle \in C$ such that $p_i = def_i(s \downarrow_{con_i})$ and $\exists \langle def_j, con_j \rangle \in C$ $n_j = def(s \downarrow_{con_j})$. A solution s is optimal if there is no other solution s' with $pref(s') > pref(s)$. In this definition, the preference of a solution s is obtained by combining all the positive preferences associated to its projections over the constraints, combining all the negative preferences associated to its projections over the constraints, and then, combining the two preferences obtained so far. This definition avoids problems due to non-associativity of $\times$.

2.3 Fuzzy preferences and uncertainty

In this section we present a formalism for dealing with fuzzy preference problems, in which there are some uncontrollable variables defined by a possibility distribution [12].

Possibility theory. Possibility theory was introduced in [13], in connection with the fuzzy set theory, to allow reasoning to be carried out on imprecise or vague knowledge, making it possible to deal with uncertainties on this knowledge. This theory and its developments constitute a method of formalizing non-probabilistic uncertainties on events, i.e., a way of assessing to what extent the occurrence of an event is possible and to what extent we are certain of its occurrence, without, however, knowing the evaluation of the probability of this occurrence. This can happen, for instance, when there is no similar event to be referred to. Possibility theory, represents the uncertainty on the occurence of an event in the form of possibility distributions. In what follows we will consider events represented by an uncontrollable variable taking a value from a particular subset.

A possibility distribution π_x associated to a single valued variable x with domain D is a mapping from D to a totally ordered scale L (usually $[0, 1]$) such that $\forall d \in D$, $\pi_x(d) \in L$ and $\exists\ d \in D$ such that $\pi_x(d) = 1$, where 1 the top element of the scale L. The following conventions hold: $\pi_x(d) = 0$ means $x = d$ is impossibile; $\pi_x(d) = 1$ means $x = d$ is fully possibile, unsurprizing.

A possibility distribution is similar to a probability density. However, $\pi_x(d) = 1$ only means that $x = d$ is a plausible situation, which cannot be excluded. Thus, a degree of possibility can be viewed as an upper bound of a degree of probability. Possibility theory encodes incomplete knowledge while probability accounts for random and observed phenomena. In particular, the possibility distribution π_x can encode: i) *complete ignorance* about x: $\pi_x(d) = 1, \forall d \in D$; in this case all values $d \in D$ are plausible for x and so it is impossible to exclude any of them and ii) *complete knowledge* about x: $\pi_x(\bar{d}) = 1, \exists \bar{d} \in D$ and $\pi_x(d) = 0\ \forall d \in D$, $d \neq \bar{d}$; in this case only the value $\bar{d}$ is plausible for x.

Given a possibility distribution π_x associated to a variable x, the occurrence of the event $x \in E \subseteq D$ can be defined by the possibility and the necessity degrees. The possibility degree of an event "$x \in E$", denoted by $\Pi(x \in E)$ or simply by $\Pi(E)$, is $\Pi(x \in E) = sup_{d \in E} \pi_x(d)$. It evaluates the extent to which "$x \in E$" is *possibly* true. In particular, $\Pi(x \in E) = 1$ means that the event $x \in E$ is totally possible. However it could also not happen. Therefore in this case we are completely ignorant about its occurrence. While $\Pi(x \in E) = 0$ means that the event $x \in E$ for sure will not happen.

The necessity degree of "$x \in E$", denoted by $N(x \in E)$ or simply by $N(E)$, is $N(x \in E) = inf_{d \notin E} c(\pi_x(d))$, where c is the order reversing map such that $c(p) = 1 - p$ and E^C is the complement of E in D. It evaluates the extent to which "$x \in E$" is *certainly* true. In particular, $N(x \in E) = 1$ means that the event $x \in E$ is certain, $N(x \in E) = 0$ means that the event is not necessary at all, although it may happen. In fact, $N(x \in E) = 0$ iff $P(x \in E^C) = 1$.

The possibility and the necessity measures are related by the following formula $\Pi(E) = 1 - N(E^C)$. From this, follows $N(E) = 1 - \Pi(E^C)$.

Uncertainty in soft constraints. Whereas in usual soft constraint problems all the variables are assumed to be controllable, i.e., their value can be decided according to the constraints which relate them to other variables, in many real-world problems uncertain parameters must be used. Such parameters are associated with variables which are not under the user's direct control and that can be assigned only by Nature.

In [8], these problems are formalized as a set of variables, that can be controllable and uncontrollable and a set of fuzzy constraints linking these variables. More precisely, an uncertain soft constraint satisfaction problem (USCSP) is defined by a tuple $\langle S, V_c, V_u, C\rangle$, where S is a semiring, V_c is the set of controllable variables, V_u is the set of uncontrollable variables, and C is the set of soft constraints. The soft constraints in C may involve any subset of variables of $V_c \cup V_u$. While in a classical soft constraint problem we can decide how to assign the variables to make the assignment optimal, in the presence of uncertain parameters we must assign values to the controllable variables guessing what Nature will do with the uncontrollable variables. A solution in USCSP is an assignment to all its controllable variables. Depending on the assumptions made on the observability of the uncontrollable variables, different optimality criteria can be defined. For example, an optimal solution for an USCSP can be defined as an assignment of values to the variables in V_c such that, *whatever* Nature will decide for the variables in V_u, the overall assignment will be optimal. This corresponds to assume that the values of the uncontrollable variables are never observable, i.e., that the values of the controllable variables are decided upon without observing the values of the uncontrollable variables. This is a pessimistic view, and, often, an assignment satisfying such a requirement does not exist. In such a case, one can relax the optimality condition to that of having a preference above a certain threshold α in all scenarios. In this case solving the problem will consist of finding the assignments to variables in V_c which satisfy this property at the level that is highest α. Furthermore, one could be satisfied with finding an assignment of values to the variables in V_c such that, *for at least* one assignment decided by Nature for the variables in V_u, the overall assignment will be optimal. This definition follows an optimistic view. Other definitions can be between these two extremes.

Moreover, the uncontrollable variables can be equipped with additional information on the likelihood of their values. Such information can be given in several ways, depending on the amount and precision of knowledge we have. In this paper for expressing such information we will consider possibility distributions. This information can be used to infer new soft constraints over the controllable variables, expressing the compatibility of the controllable parts of the problem with the uncertain parameters, and can be used to change the notion of optimal solution.

In this paper we will consider the approach of guaranteeing a certain preference level α taking into account the additional information on the uncontrollable variables provided in the form of possibility distribution.

Unifying fuzzy preferences and uncertainty via possibility theory. An algorithm for solving uncertain fuzzy CSP keeping separate preference and uncertainty is Algorithm SP [12], which works as follows.

It starts from an uncertain fuzzy CSP $Q = \langle S_{FCSP}, V_c, V_u, C = C_f \cup C_{fu} \rangle$, where C_f is the set of constraints of Q defined only on controllable variables, C_{fu} is the set of constraints of Q defined on both controllable and uncontrollable variables.

It obtains a fuzzy CSP $Q' = \langle S_{FCSP}, V_c, C' = C_{control} \cup C_u \rangle$, where:

- $C_{control} = C_f \cup C_p$, where C_p is the set of constraints obtained by projecting the constraints C_{fu} on the controllable variables.
- C_u is the set of constraints, defined only on controllable variables, obtained from the constraints C_{fu} applying a specific procedure, that is described just down in this section.

Every constraint in C_u is computed as follows [8]. Consider a fuzzy constraint C, represented by the fuzzy relation R, which relates a set of controllable variables $X = \{x_1, \ldots, x_n\}$, with domains $D_1, \ldots, D_n$, to a set of uncontrollable variables $Z = \{z_1, \ldots, z_k\}$ with domains $A_1, \ldots, A_k$. Assume the knowledge of the uncontrollable variables is modeled with the possibility distribution π_Z defined on $A_Z = A_1 \times \cdots \times A_k$. Assume the preferential information is instead represented by function $\mu_R : D_X \times A_Z \longrightarrow [0, 1]$, where $D_X = D_1 \times \cdots \times D_n$. Value $\mu_R(d, a)$ is the preference associated to the assignment to controllable and uncontrollable variables $(d, a) = (d_1, \ldots, d_n, a_1, \ldots, a_k)$. The constraint C is considered satisfied[2] by assignment $d = (d_1, \ldots, d_n) \in D_1 \times \cdots \times D_n$ if, *whatever the values of* $a = (a_1, \ldots, a_k)$*, these values are compatible*[3] *with* d, i.e., if the set of possible values for z is included in $T = \{a \in A_Z | \mu_R(d, a) > 0\}$. Given assignment $d \in D_X$, and $\mu_T(a) = \mu_R(d, a)$, the preference of d in the new constraint C' obtained from C removing uncontrollable variables is:

$$\mu'(d) = N(d\, satisfies\, C) = N(z \in T) = inf_{a \in A_Z} max(\mu_T(a), c(\pi_Z(a))) \quad (1)$$

where c is the order map such that $c(p) = 1 - p$, $\forall p \in [0, 1]$. The value $\mu'(d)$, that is given by the necessity degree of the event "d satisfies C", represents the degree of satisfaction of C. It is characterized by the following property: $\mu'(d) \geq \alpha$ iff when $\pi_Z(a) > c(\alpha)$ then $\mu_R(d, a) \geq \alpha$, where a is the actual value of z. Informally, the new preference level of the assignment d obtained reasoning on uncertainty, $\mu'(d)$, is greater or equal than α if and only if the assignments ($X = d, Z = a$), such that the possibility $\pi_Z(a)$ is strictly greater than $1 - \alpha$, had a preference $\mu_R(d, a)$ greater or equal than α in the original problem.

For each complete assignment s to V_c, we compute $F(s)$, $P(s)$ and $U(s)$, that are respectively the minimum preference over the constraints in C_f, C_p and C_u. Finally, the preference of every complete assignment is given by $F_P(s) = min(F(s), P(s))$ and U.

Once each solution is associated with two values, the satisfaction degree F_P and the robustness U, there are various approaches for ordering solutions which differ on the attitude toward risk they implement.

[2] Here "satisfied" means "at least partially satisfied".

[3] A value a is compatible with d if $\mu_R(d, a) > 0$.

Given an USCSP Q, consider a solution s with corresponding satisfaction degree $F_P(s)$ and robustness $U(s)$. Each semantics associates to s the ordered pair $\langle a_s, b_s\rangle$ as follows. *Risky* (R) and *Diplomatic* (D) associate to s the pair $\langle a_s, b_s\rangle$=$\langle F_P(s), U(s)\rangle$ and *Safe* (S) the pair $\langle a_s, b_s\rangle$=$\langle U(s), F_P(s)\rangle$.

Given two solutions s_1 and s_2, let $\langle a_1, b_1\rangle$ and $\langle a_2, b_2\rangle$ represent the pairs associated to the solutions by each semantics in turn, then Risky, Safe and Diplomatic semantics work as follows.

– *Risky* and *Safe*. If $a_1 > a_2$ then $\langle a_1, b_1\rangle >_J \langle a_2, b_2\rangle$ (and the opposite for $a_2 > a_1$). If $a_1 = a_2$ then, if $b_1 > b_2$ then $\langle a_1, b_1\rangle >_J \langle a_2, b_2\rangle$ (and the opposite for $b_2 > b_1$), while if $b_1 = b_2$ then $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$, where $J = R, S$
– *Diplomatic*. If $a_1 \leq a_2$ and $b_1 \leq b_2$ then $\langle a_1, b_1\rangle \leq_D \langle a_2, b_2\rangle$ (and the opposite for $a_2 \leq a_1$ and $b_2 \leq b_1$), if $a_1 = a_2$ and $b_1 = b_2$ then $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$, else $\langle a_1, b_1\rangle \bowtie \langle a_2, b_2\rangle$ ($\bowtie$ means incomparable).

All semantics, except Diplomatic, can be regarded as a Lex ordering on pairs $\langle a_s, b_s\rangle$ with the first component as the most important feature. Diplomatic, instead, is a Pareto ordering on the pairs. The first semantics, called *Risky*, considers F_P as the most important feature. Informally, the idea is to give more relevance to the satisfaction degree that can be reached in the best case considering less important a high risk of being inconsistent. Hence we are risky, since we disregard almost completely the uncertain part of the problem. The second semantics, called *Safe*, represents the opposite attitude with the respect to the previous one, since it considers $U(s)$ as the most important feature. Informally, the idea is to give more importance to the robustness level that can be reached considering less important having a high preference. In particular, in this case we consider a solution better than another one if its robustness is higher, i.e., if it guarantees an higher number of scenarios with an higher preference. The last semantics, called *Diplomatic*, aims at giving the same importance to the two aspects of a solution: satisfaction degree and robustness. As mentioned above, the Pareto ordering on pairs $\langle a_s, b_s\rangle$ is adopted. The idea is that a pair is to be preferred to another only if it wins both on preference and robustness, leaving incomparable all the pairs that have one component higher and the other lower. Contrarily to the Diplomatic semantics, the other semantics produce a total order over the solutions.

3 Uncertainty in bipolar preference problems

In this section we define bipolar preference problems with uncertainty and we give an algorithm for translating them in a new kind of bipolar preference problems without uncertainty. Then we give a way for computing the preference of a solution of these new problems and we show that the same semantics mentioned in the previous section can be used here for ordering the solutions.

Uncertain bipolar preference problems are problems that are characterized by a set of variables, which can be controllable or uncontrollable, and by a set of bipolar constraints (see Section 2.2). The domain of every uncontrollable variable is equipped with a possibility distribution, that specifies, for every value in the domain, the degree of plausibility that the variable takes that value.

Definition 1 (UBCSP). *An uncertain bipolar CSP (UBCSP) is a tuple* $\langle bS, V_c, V_u, bC = bC_f \cup bC_{fu}\rangle$*, where*

- $bS = (N, P, +, \times, \bot, \Box, \top)$ *is a bipolar preference structure;*
- $V_c = \{x_1, \dots x_n\}$ *is the set of controllable variables,*
- $V_u = \{z_1, \dots z_k\}$ *is the set of uncontrollable variables with possibility distributions* $\{\pi_1, \dots \pi_k\}$*,*
- $bC = bC_f \cup bC_{fu}$ *is the set of bipolar constraints, that may involve any subset of variables of* $V_c \cup V_u$*. More precisely, constraints in* bC_f *involve only on a subset of controllable variables of* V_c*, while constraints in* bC_{fu} *involve both a subset of variables of* V_c *and a subset of variables in* V_u*.*

3.1 Removing uncertainty from UBCSPs

We now describe an algorithm, that we call *B-SP*, for handling with UBCSPs, that generalizes algorithm *SP* described in Section 2.3 for fuzzy preferences, to the case of positive and negative totally ordered preferences. This algorithm takes in input an uncertain bipolar preference problem $BQ = \langle bS, V_c, V_u, BC = BC_f \cup BC_{fu}\rangle$, where $bS = (N, P, +, \times, \bot, \Box, \top)$, N and P are totally ordered sets w.r.t. the ordering induced by $+$ and it returns a new kind of bipolar preference problem without uncertainty. It is mainly characterized by two steps: in the first one it transforms the given UBCSP in a new kind of bipolar problem with uncertainty, in order to be able to handle separately the positive and the negative preferences, and in the second one it removes uncertainty from this problem.

1st step: translation into a new kind of UBSCP Since we are not assuming that the compensation operator $\times$ of bS is associative, then, for avoiding problems due to non-associativity, we translate the given UBCSP into a new kind of bipolar preference problem, that allows to handle separately the positive and the negative preferences of BQ. In order to get this, we introduce 2-bipolar constraints, that are similar to bipolar constraints, except that they associate to each assignment not a unique (positive or negative) value, but a pair of values, that is, a positive and a negative one. We consider also 2-bipolar CSPs, that are just a set of variables and a set of 2-bipolar constraints over these variables.

The first step of *B-SP* regards the translation of every constraint $Bc = \langle \mu, con\rangle$ in BC into a corresponding 2-bipolar constraint $bc = \langle b\mu, con\rangle$ as follows. For every assignment d to variables in con, if $\mu(d) \in P$, then $b\mu(d) = (\mu(d), \Box)$, whereas if $\mu(d) \in N$, then $b\mu(d) = (\Box, \mu(d))$, i.e., if the starting preference of d is positive, then we put that preference in the first component of the pair, and indifference in the other component, otherwise, we put starting negative preference in the second component of the pair and indifference in the other one.

Doing so for every constraint of bC, we obtain an uncertain 2-bipolar CSP $bQ = \langle bS, V_c, V_u, bC = bC_f \cup bC_{fu}\rangle$, which is like the uncertain bipolar preference problem BQ except that every constraints resp. in BC_f, and BC_{fu} is translated in the corresponding 2-bipolar constraint resp. in bC_f and bC_{fu}. Since now bQ is a problem with uncertainty that keep separate positive and negative preferences, then we can reason separately with these two kinds of preferences.

2nd step: elimination of uncertainty. The next step is characterized by the translation of the 2-bipolar CSP bQ with uncertainty in a 2-bipolar CSP without uncertainty $bQ' = \langle bS, V_c, bC' = bC_f \cup bC_p \cup bC_u \rangle$. This is obtained by eliminating the uncontrollable variables and the 2-bipolar constraints in bC_{fu} relating controllable and uncontrollable variables and by adding new 2-bipolar constraints only among these controllable variables. These new constraints, that can be classifi ed in two sets of constraints, that we call bC_u and bC_p, generalize the constraints in C_u and C_p computed by *SP*. We recall that in *SP* constraints in C_u are obtained by applying a specifi c procedure for removing uncontrollability and constraints in C_p are computed for recalling the best preference that can be obtained in the removed constraints.

Constraints in bC_u. Every 2-bipolar constraint $bc = \langle b\mu, con \rangle$ in bC_{fu}, i.e. such that $con \cap V_c = X$ and $con \cap V_u = Z$ is translated into a 2-bipolar constraint $bc' = \langle b\mu', con' \rangle$ in bC_u, where $con' = X$, such that for every assignment (d, a) to $X \times Z$, with $b\mu(d, a) = (b\mu_{pos}(d, a), b\mu_{neg}(d, a))$, $b\mu'(d) = (b\mu'_{pos}(d), b\mu'_{neg}(d))$, where $b\mu'_{pos}(d)$ and $b\mu'_{neg}(d)$ are obtained by applying a formula similar to the one presented in Section 2.3 considering resp. $b\mu_{pos}(d, a)$ and $b\mu_{neg}(d, a)$ instead of $\mu(d, a)$.

Recall that in *SP* every constraint $\langle \mu, con \rangle$ in C_{fu}, i.e. such that $con \cap V_c = X$ and $con \cap V_u = Z$, is translated in a constraint $\langle \mu', con' \rangle$ in C_u, where $con' = X$ and for every assignment d to X, μ' is defi ned as follows [8]: $\mu'(d) = inf_{a \in A_Z} max(\mu(d, a), c(\pi_Z(a)))$, where c is the order reversing map in $[0, 1]$ such that $c(p) = 1 - p$ and where π_Z is the possibility distribution of Z, which has domain A_Z. This defi nition depends on the assumption of commensurability between preferences and possibilities, that can be done since fuzzy preferences and possibilities are defi ned in the same scale (i.e., in $[0, 1]$). It depends also on the fact that the maximum operator is the additive operator of the fuzzy c-semiring and on the fact that c is an order reversing map in $[0, 1]$ w.r.t. the ordering induced by the maximum operator such that $c(c(p)) = p, \forall p \in [0, 1]$.

Since we want to use a similar formula for both positive and negative preferences, but the set of positive and negative preferences, i.e., P and N, are not necessarly the interval $[0, 1]$, we propose to map in $[0, 1]$ the positive and the negative preferences of every assignment $(d, a) \in X \times Z$ in every constraint $bc \in bC_{fu}$. We perform this mapping via functions, that we call resp. f_p and f_n, that are stricly monotone functions w.r.t. the ordering $\leq_S$ induced by the operator $+$ of bS. More precisely, if $P = [a, b]$ (resp. $N = [a, b]$) with $a < b$, then f_p (resp. f_n): $[a, b] \to [0, 1]$ associates to every $x \in [a, b]$ the value $\frac{x+|a|}{b+|a|} \in [0, 1]$. Then we can apply the formula recalled above, by replacing the maximum operator with operator $+$ of bS, the map c with a map c_S which reverses the ordering in $[0, 1]$ w.r.t. the ordering $\leq_S$ induced by $+$ of b_S and by assuming that operator inf applied to a set A returns the worst element of A w.r.t the ordering $\leq_S$. Since all the other preferences in the problems are in P and N, then we map again in P and N the values returned by the formula, by using resp. the inverse functions f_p^{-1} and f_n^{-1}. f_p^{-1} (resp. f_n^{-1}): $[0, 1] \to [a, b]$ associates to every $y \in [0, 1]$ the value $[y(b + |a|) - |a|] \in [a, b]$. Notice that the fact that f_p and f_n are stricly monotone function w.r.t. the ordering $\leq_S$ induced by the operator $+$ of bS, implies that they are invertible and their inverse functions are monotone w.r.t. the same ordering.

More formally, we build the set bC_u from bC_{fu} as follows.

1. Every 2-bipolar constraint $bc = \langle b\mu, con\rangle$ in bC_{fu} such that $con \cap V_c = X$ and $con \cap V_u = Z$, is translated in a 2-bipolar constraint with preferences in $[0,1]$, $bc^* = \langle b\mu^*, con\rangle$, where, for every assignment (d,a) to $X \times Z$, $b\mu^*(d,a) = (b\mu^*_{pos}(d,a), b\mu^*_{neg}(d,a))$ and $b\mu^*_{pos}(d,a) = f_p(b\mu_{pos}(d,a)) \in [0,1]$ and $b\mu^*_{neg}(d, a) = f_n(b\mu_{neg}(d,a)) \in [0,1]$.
2. Then bc^* is translated into the 2-bipolar constraint $bc^{*'} = \langle b\mu^{*'}, con^{*'} = X\rangle$, only on controllable variables, where for every assignment d to X, $b\mu^{*'}(d) = (b\mu^{*'}_{pos}(d), b\mu^{*'}_{neg}(d))$ and $b\mu^{*'}_{pos}(d)$ and $b\mu^{*'}_{neg}(d)$ are computed by following the a procedure similar to the one described above, that is, $b\mu^{*'}_{pos}(d) = inf_{a\in A_Z}(b\mu^*_{pos}(d, a) + c_S(\pi_Z(a)))$ and $b\mu^{*'}_{neg}(d) = inf_{a\in A_Z}(b\mu^*_{neg}(d,a) + c_S(\pi_Z(a)))$, where c_S is an order reversing map w.r.t. $\leq_S$ in $[0,1]$, such that $c_S(c_S(p)) = p$.
3. Finally, $bc^{*'}$ is translated in a new 2-bipolar constraint $bc' = \langle b\mu', con' = X\rangle$ in bC_u where for every assignment d to X, $b\mu'(d) = (b\mu'_{pos}(d), b\mu'_{neg}(d)) \in P \times N$, where $b\mu'_{pos}(d) = f_p^{-1}(b\mu^{*'}_{pos}(d))$ and $\mu'_{neg}(d) = f_n^{-1}(b\mu^{*'}_{neg}(d))$.

Notice that the property described in Section 2.3 characterizing the preference function μ' of every constraint in C_u (i.e., $\mu'(d) \geq \alpha$ iff when $\pi_Z(a) > c(\alpha)$ then $\mu(d,a) \geq \alpha$, where a is the actual value of z and c is the order reversing map in $[0,1]$ s.t. $c(p) = 1 - p$) holds also in our framework for both $b\mu'_{pos}(d)$ and $b\mu'_{neg}(d)$ as follows:

– $b\mu'_{pos}(d) \geq_S \beta \in P$ iff when $\pi_Z(a) > c_S(f_p(\beta))$, then $b\mu_{pos}(d,a) \geq_S \beta$;
– $b\mu'_{neg}(d) \geq_S \alpha \in N$ iff when $\pi_Z(a) > c_S(f_n(\alpha))$, then $b\mu_{neg}(d,a) \geq_S \alpha$.

Notice also that the procedure above for removing uncontrollability holds both for positive and negative preferences, since it is not based on the combination operators ($\times_p$ and $\times_n$) of positive and negative preferences, which have different behaviours.

Constraints in bC_p. Constraints in bC_p generalize constraints in C_p of *SP*. Recall that constraints in C_p are added to the resulting problem without uncertainty, in order to avoid having solutions with satisfaction degree F strictly better than the best one in the original problem. In the case of fuzzy preferences, adding these constraints is useful, since the aggregation of fuzzy preferences goes down in the ordering. This is also reasonable for the negative preferences whose combination follows the same behaviour. For the positive preferences, instead, where the combination goes up in the ordering, is reasonable to save the worst positive preference obtained in the original problem, in order to avoid to give a solution with positive degree of satisfaction that is strictly lower than the ones that can be effectively obtained.

Hence, we define the set of constraints bC_p as follows. Given a 2-bipolar constraint $bc =< b\mu, con >$ in bC_{fu}, such that $con \cap V_c = X$ and $con \cap V_u = Z$, then the corresponding 2-bipolar constraint in bC_p is $bc_p =< b\mu_p, con_p = X >$, and μ_p is such that for every assignment d to X, $b\mu_p(d) = (b\mu_{p_{pos}}(d), b\mu_{p_{neg}}(d)) \in P \times N$, where $b\mu_{p_{neg}}(d)$ (resp. $b\mu_{p_{pos}}(d)$) is the best negative (resp. the worst positive) preference that can be reached for d in bc when we consider the various values a in the domain of the uncontrollable variables in con, i.e., $b\mu_{p_{neg}}(d) = \sum_{n_{\{a\in A_Z\}}} b\mu_{neg}(d,a)$ and $b\mu_{p_{pos}}(d) = \inf_{p_{\{a\in A_Z\}}} b\mu_{pos}(d,a)$, where A_Z is the domain of Z, $\sum_n$ is the operator $+_n$ of the negative preferences applied to more than two negative preferences that returns the best negative preference and inf_p is the operator that, applied to a set of positive preferences, returns its worst positive preference w.r.t ordering induced by $+_p$.

3.2 Solutions ordering

We solve the problem without uncertainty bQ' returned by *B-SP* by associating to each solution both a degree of satisfaction and a degree of robustness. More precisely, for every solution s of bQ', i.e. for every complete assignment to V_c, we compute $F_{pos}(s)$, $P_{pos}(s)$, $U_{pos}(s)$, that are resp. obtained by combining, via operator $\times_p$, all the positive preferences of the projections of s over the constraints in bC_f, bC_p and bC_u, and $F_{neg}(s)$, $P_{neg}(s)$, $U_{neg}(s)$, that are resp. obtained by combining, via operator $\times_n$, all the negative preferences of the projections of s over the constraints in bC_f, bC_p and bC_u. Hence, we compute two satisfaction levels, a positive one, i.e., $F_{P_{pos}}(s) = F_{pos}(s) \times_p P_{pos}(s)$ and a negative one, i.e., $F_{P_{neg}}(s) = F_{neg}(s) \times_n P_{neg}(s)$ and two degrees of robustness, i.e., U_{pos} and U_{neg}, that characterize resp. the positive and the negative satisfaction degree. Then we can compensate the two degrees of satisfactions and the two degrees of robustness. Hence, we associate to every solution a degree of satisfaction $F_P(s) = F_{P_{pos}}(s) \times F_{P_{neg}}(s)$ and a robustness degree $U(s) = U_{pos}(s) \times U_{neg}(s)$. Since every solution is associated to a pair composed by a satisfaction degree and a robustness degree, in order to compare solutions, we can use the same semantics (i.e., Risky, Safe and Diplomatic) described in Section 2.3.

Notice that the derived properties presented in [12] continue to hold. The first one states that, if we fix possibilities of uncontrollable variables and if we increase preferences of a given assignment in a constraint in C_{fu} to controllable variables and uncontrollable variables for every value in the domain of the uncontrollable variables, then we obtain a higher value of robustness. This property holds also in our more general framework where the robustness of a solution is given by the compensation between U_{pos} and U_{neg}. In fact, if we increase preferences of that assignment to controllable variables, then we obtain values U'_{pos} and U'_{neg} which are higher than U_{pos} and U_{neg}, since we assume that functions f_p and f_n mapping resp. positive and negative preferences in $[0, 1]$ are monotone and since $\times_p$ and $\times_n$ are monotone. Moreover, by monotonicity of the compensation operator, if U'_{pos} and U'_{neg} increase, then also robustness U' which is the combination of U'_{pos} and U'_{neg} increases.

The other property in [12] states that if we fix preferences in a constraint in C_{fu} and if we decreases possibilities of uncontrollable variables, then we obtain an higher robustness. This continues to hold also in our scenario for both U_{pos} and U_{neg} since the proof of this property is based only on the fact that c_S is an order reversing map w.r.t. $\leq_S$, i.e., if $a_1 \leq a_2$ then $c_S(\pi_1) \geq_S c_S(\pi_2)$, and on the fact that the combination of positive preferences and the combination of negative preferences are monotone w.r.t. $\leq_S$. We can conclude as above. Since the compensation operator is monotone then also our robustness (that is, $U = U_{pos} \times U_{neg}$), increases.

4 Example

Figure 1 shows an example of an uncertain bipolar CSP, that we call BQ, which is defined by $\langle bS, V_c = \{x, y\}, V_u = \{z_1, z_2\}, BC = BC_f \cup BC_{fu}\rangle$. The bipolar structure

bS is ($N = [-1, 0]$, $P = [0, 1]$, $+ = max$, $\times$, $\bot = -1$, $\Box = 0$, $\top = 1$), where $\times$ is s. t. $\times_p = max$, $\times_n = min$ and $\times_{np} = sum$. The set of constraints bC_{fu} contains $c1 = \langle \mu_1, \{x, z_1\} \rangle$ and $c2 = \langle \mu_2, \{x, z_2\} \rangle$, while bC_f contains $c3 = \langle \mu_3, \{x, y\} \rangle$. Figure 1 shows the positive and the negative preferences within such constraints, as well as the possibility distributions π_1 and π_2 over domains of z_1 and z_2.

Figure 2 (a) shows the uncertain 2-bipolar CSP $bQ = \langle bS, V_c = \{x, y\}, V_u = \{z_1, z_2\}, bC = bC_f \cup bC_{fu} \rangle$ built in the 1st step of *B-SP*. Figure 2 (b) shows the 2-bipolar CSP $bQ' = \langle bS, V_c = \{x, y\}, bC' = bC_f \cup bC_p \cup bC_u \rangle$, built in the 2nd step of *B-SP*. bC_f is composed by $c3 = \langle \mu_3, \{x, y\} \rangle$, bC_p by $cp1 = \langle \mu_{p1}, \{x\} \rangle$ and $cp2 = \langle \mu_{p2}, \{x\} \rangle$ and bC_u by $c1' = \langle \mu'_1, \{x\} \rangle$ and $c2' = \langle \mu'_2, \{x\} \rangle$. $c1'$ and $c2'$ are obtained by using functions $f_n : N = [-1, 0] \rightarrow [0, 1]$ mapping every value $n \in [-1, 0]$ into the value $(n + 1) \in [0, 1]$, $f_n^{-1} : [0, 1] \rightarrow [-1, 0]$ mapping every value $t \in [0, 1]$ into the value $(t - 1) \in [-1, 0]$ and c_S mapping every $p \in [0, 1]$ in $1 - p$.

Figure 2 (c) shows all the solutions of our UBCSP BQ that are complete assignments to all the controllable variables (thus x and y). To compute the preference of a solution s, we need the positive satisfaction degree $F_{P_{pos}}(s)$ (resp., the negative satisfaction degree $F_{P_{neg}}(s)$) obtained by combining via $\times_p = max$ (resp., $\times_n = min$) all the positive preferences associated to the projections of s in constraints on $bC_f \cup bC_p$, that are $c3$, $cp1$ and $cp2$. Then we compute the positive robustness $U_{pos}(s)$ (resp., the negative robustness $U_{neg}(s)$) by combining via $\times_p = max$ (resp., $\times_n = min$) all the positive preferences associated to the projections of s in constraints in bC_u, i.e., in this case in $c1'$ and $c2'$. Then we obtain a unique satisfaction degree $F_P(s)$ for s by compensating (via $\times_{np} = sum$) $F_{P_{pos}}(s)$ and $F_{Pneg}(s)$ and a unique robustness value by compensating $U_{pos}(s)$ and $U_{neg}(s)$.

The optimal solution for the Risky semantics is $s_3 = (y = b, x = a)$, which has preference $(F_P = 0.5, U = -0, 2)$; for the Safe semantics we have $s_4 = (y = b, x = b)$, which has preference $(F_P = 0.3, U = 0.1)$. For the Diplomatic semantics s_3 and s_4 are equally optimal. Notice that the solutions chosen by the various semantics differ on the attitude toward risk. In particular, the Risky semantics is risky, since it disregards almost completely the uncertain part of the problem. In fact, in this example it chooses the solution that gives an high positive preference in the controllable part, even if the uncontrollable part, which must be decided by Nature, will give with high possibility a negative preference. On the other hand, for the Safe semantics is better to select the solution with a higher robustness, i.e., that guarantees a higher number of scenarios with a higher preference. In this example, Safe chooses a solution with a lower preference w.r.t. Risky, but that will have with high possibility a positive preference in the uncontrollable part.

5 Related and future work

Both bipolar reasoning and preferences, and preferences and uncertainty have recently attracted some interest in the AI community. In [1] a bipolar preference model based on a fuzzy-possibilistic approach is described where fuzzy preferences are considered and negative preferences are interpreted as violations of constraints. In particular, precedence is given to negative preference optimization, and positive preferences are only

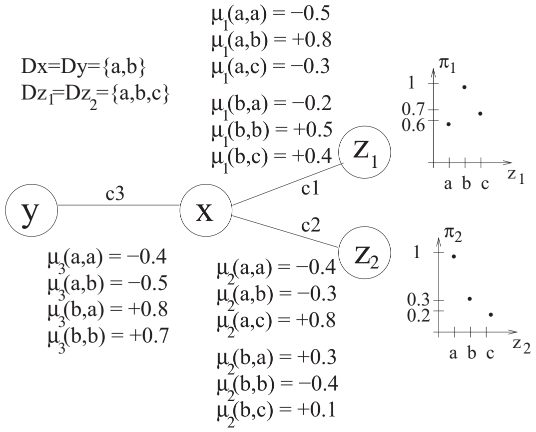
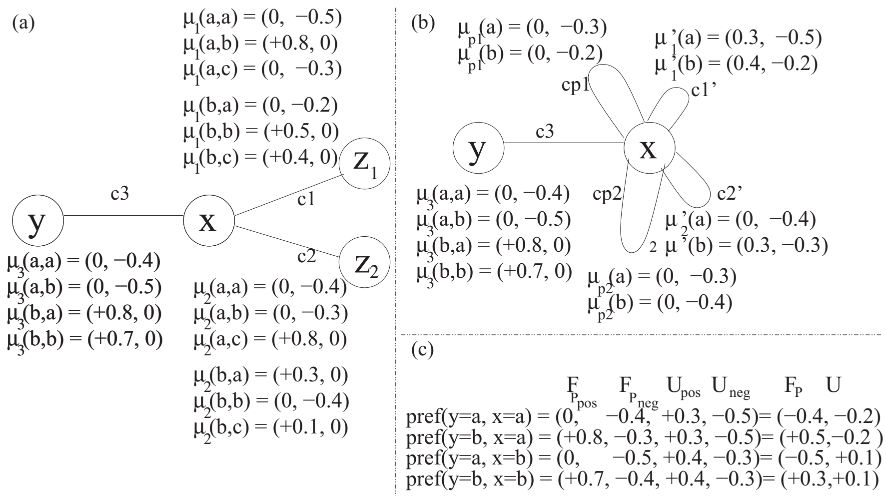


Figure1. An uncertain bipolar CSP.



	$F_{P_{pos}}$	$F_{P_{neg}}$	U_{pos}	U_{neg}	F_P	U
pref(y=a, x=a) =	(0,	−0.4,	+0.3,	−0.5)=	(−0.4,	−0.2)
pref(y=b, x=a) =	(+0.8,	−0.3,	+0.3,	−0.5)=	(+0.5,	−0.2)
pref(y=a, x=b) =	(0,	−0.5,	+0.4,	−0.3)=	(−0.5,	+0.1)
pref(y=b, x=b) =	(+0.7,	−0.4,	+0.4,	−0.3)=	(+0.3,	+0.1)

Figure2. How algorithm *B-SP* works on the UBSCP of Figure 1.

used to distinguish among the optimals found in the first phase, thus not allowing for compensation. In [11] bipolar preference scales are considered and it is defined an operator, the $uninorm$, which can be seen as a restricted form of compensation but that is forced to always be associative. Our work deals also with uncertainty and with compensation operators that can be non-associative. In [4, 5, 7, 9] have been defined many approaches with one kind of preferences and uncertainty, but they don't mix these two aspects, hence they can't compare directly preferred assignments and uncertain events.

In this paper we have proposed an algorithm for handling with problems with both positive and negative preferences, and with some uncontrollable variables, which model the uncertainty of the problem. After having defined such problems, we have proposed to handle them by extending existing techniques to handle bipolar problems and problems with uncertainty. We plan to adapt constraint propagation and branch and bound techniques to deal with uncertain bipolar problems and we intend to develop a solver for uncertain bipolar CSPs, which should be flexible enough to accommodate for both associative and non-associative compensation operators.

Acnowledgements

This work has been supported by Italian MIUR PRIN project "Constraints and Preferences" (n. 2005015491).

References

1. S. Benferhat, D. Dubois, S. Kaci, and H. Prade. Bipolar possibility theory in preference modeling: representation, fusion and optimal solutions. *Information Fusion*, 2006.
2. S. Bistarelli, U. Montanari, and F. Rossi. Semiring-based constraint solving and optimization. *Journal of the ACM*, 44(2):201–236, mar 1997.
3. S. Bistarelli, M. S. Pini, F. Rossi, and K. B. Venable. Bipolar preference problems. In *ECAI-06 (poster)*, 2006.
4. R. I. Brafman and M. Tennenholtz. On the foundations of qualitative decision theory. In *AAAI'96*, pages 1291–1296, 1996.
5. R. I. Brafman and M. Tennenholtz. On the axiomatization of qualitative decision criteria. In *AAAI'97*, 1997.
6. R. Dechter. *Constraint processing*. Morgan Kaufmann, 2003.
7. D. Dubois, H. Fargier, and P. Perny. On the limitations of ordinal approaches to decision making. In *KR 2002*, pages 133–144, 2002.
8. D. Dubois, H. Fargier, and H. Prade. Possibility theory in constraint satisfaction problems: Handling priority, preference and uncertainty. *Appl. Intell.*, 6(4):287–309, 1996.
9. H. Fargier and R. Sabbadin. Qualitative decision under uncertainty: back to expected utility. In *IJCAI'03*, pages 303–308, 2003.
10. H. Fargier, T. Schiex, and G. Verfaille. Valued Constraint Satisfaction Problems: Hard and Easy Problems. In *IJCAI-95*, pages 631–637. Morgan Kaufmann, 1995.
11. M. Grabisch, B. de Baets, and J. Fodor. The quest for rings on bipolar scales. *Int. Journ. of Uncertainty, Fuzziness and Knowledge-Based Systems*, 2003.
12. M. S. Pini, F. Rossi, and K. B. Venable. Possibility theory for reasoning about uncertain soft constraints. In *ECSQARU-05*, volume 3571 of *LNCS*. Springer, 2005.
13. L.A. Zadeh. Fuzzy sets as a basis for the theory of possibility. *Fuzzy sets and systems*, pages 13–28, 1978.