Report on the PhD thesis of Maria Silvia Pini "Reasoning with preferences and uncertainty"

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The concept of a preference is often used, both in Artificial Intelligence and Economics. However, in each discipline different aspects have been addressed and different results have been aimed for. This presents a potential for an interdisciplinary research but at the same time requires a proper knowledge of the relevant literature and of the objectives of both disciplines. The candidate succeeded in this endevour admirably by producing a PhD thesis that contains a rich variety of results concerned with different ways of formalizing and reasoning about preferences. While studying this thesis I was naturally brought towards possible suggestions for future research that I explicitly formulated. This is just one indication how fruitful such an interdisciplinary research might be.

The submitted PhD thesis consists of 4 long chapters. Each deals with a different aspect of preferences.

Chapter 2 is devoted to a detailed analysis of fuzzy preferences and uncertainties as defined through Zadeh's possibility theory. This leads to a study of uncertain fuzzy constraint satisfaction problems, in particular to an enlightening analysis of various possible semantics.

The strong point of this chapter is that an extended example is discussed and that algorithms are introduced to find optimal solutions of an uncertain fuzzy CSP w.r.t. the introduced semantics. A minor point that should be noted is that the qualification "finding efficiently" used in introducing the algorithms is not clarified (and probably cannot be). In fact, some problems solvable by means of the branch and bound method (like the knapsack problem) are NP-complete.

Chapter 3 is devoted to a study of so-called bipolar preferences according to which one expresses both positive and negative preferences. The systematic analysis begins with a definition of a bipolar preference structure and

the analysis of the behaviour of the crucial compensation operator $\times$. The concept of bipolar preferences naturally leads to that of a bipolar CSP which is the focus of the subsequent considerations. The author discusses a branch and bound algorithm for solving such CSPs and points out that in the setting of such CSPs the crucial technique of constraint propagation needs to be reconsidered before incorporating it into the algorithm.

As pointed out, the concept of bipolar preferences attracted some attention in the recent literature in AI. It might be interesting to note that this concept was also recently studied in the area of voting theory. In fact, Steven Brams considered it, though in a different setting, in his March 2006 paper *Voting Systems That Combine Approval and Preference*, written jointly with R. Sanver. This work deals with an analysis of a voting method according to which each voter submits a set of candidates he approves and a set of candidates he disapproves.

Chapter 4 is probably most interesting because of the provided generalizations of the famous results established by economists (two of which are Nobel prize winners). More specifically, the chapter includes generalizations of three celebrated theorems, those of Arrow, of Sen, and of Gibbard and Satterthwaite, that deal with the question how to aggregate these preferences so that either a common ordering or a winner is determined.

Arrow's theorem shows that, assuming certain natural assumptions on the aggregation function, the only possible aggregation function is the 'dictatorial' one that always selects the preference ordering of one of the agents. In turn, Sen's theorem shows that the most natural aggregation method, namely the 'majority rule', satisfies the assumptions of Arrow's theorem under the assumption of so-called triplewise value-restriction. Finally, the Gibbard and Satterthwaite theorem shows that under natural assumption it is not possible to construct a winner selection procedure that is immune to 'strategizing' (i.e., in plain words, 'cheating').

These results deal with the setting according to which the agents preferences are linear orderings, possibly with ties. This choice of preferences is common in economics but it is surprising that the generalization of these results to partial orderings has not been tried (though, as the author points out, some incomplete generalizations have been considered). One of the explanations could be that it is traditional in Economics to map the preferences to the orderings on reals.

The generalizations of the above results are non-trivial. First of all, they require a proper modification of the underlying notions. In particular the

notion of a dictator is not anymore unique and three versions arise. These considerations lead to generalizations of the original proofs that are not immediate. I also believe that not all proofs of the original results generalize. So the author had first to find the appropriate proofs that might generalize to partial orders.

In the process of generalization the assumption that ties are allowed was dropped. Ties are for example allowed in the proof of Arrow's theorem given by Geanakoplos. It would be interesting to see if these generalizations can also be established under the assumption that the agents preferences are preorders (i.e., partial orders with ties allowed).

The last chapter deals with incomplete orderings. In such an ordering for some pairs of elements it is not known what is the relation between them. So an incomplete ordering can be completed in a number of ways to a partial ordering with ties. It is an interesting concept that to my knowledge has not been studied in Economics. The author then studies the well-known in social choice theory procedures of a Single Transferable Vote and sequential majority voting that allow one to select an election winner. More specifically, she considers the computational complexity of the problem of computing possible and necessary winners in the case when each player submits an incomplete ordering of the candidates. She also establishes results concerned with the tractable cases.

While reading this interesting chapter it came to my mind that a possible follow up to this research might be to study incomplete preferences in the context of strategic games. In fact, the notion of incomplete information is a well-understood topic in game theory (and dealt with by means of Bayesian games) but to my knowledge games with incomplete preferences have not been analyzed.

In summary, I would like to congratulate the candidate with the obtained results. I consider it an excellent PhD thesis and, as I tried to point out, some of the problems studied and solved by the author, naturally suggest further research in the social choice theory and game theory. This indicates that the thesis is an inspiring piece of research and rich in ideas.

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