Using the Global Constraint Seeker for Learning Structured Constraint Models: A First Attempt

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Points to Remember

- Learning constraint models from positive and negative examples
- Start with **vector** of values
- Group into **regular pattern**
- Find constraint pattern that apply on group elements
- Using ***Constraint Seeker*** for *Global Constraint Catalog*
- Works for **highly structured** problems

Outline

Learning Constraint Models

- Constraint models can be hard to write
- Can we generate them automatically?
- User gives example solutions and non-solutions
- System suggests **compact conjunctions of constraints**
- User accepts/rejects constraints and/or gives more samples

Constraint Acquisition

- Active research area over last ten years
- Version space learning from AI
- Does not scale for non-binary constraints

Outline

Global Constraint Catalog

- Large collection of global constraints from literature
- Developed over the last **10 years** by **SICS** and **EMN**
- 364 constraints described (**meta data**+text) on 3000 pages
- Formal description of constraints available (*arguments + semantic: graph, logic, automata*)
- 280 constraints have executable specification

Constraint Seeker

- **CP 2011 paper** by Beldiceanu and Simonis
- How to find a constraint in catalog from examples
- Describe what the constraint should do (ground instances)
- System finds **ranked list** of potential candidate constraints
- **On-line tool** at `http://seeker.mines-nantes.fr/`

Learning Process

- Start with **flat** sample
- Group variables in **systematic** way
- Generate instances of constraints
- Find potential constraint pattern
- **Rank** by relevance
- **Remove** implied pattern by dominance checker

Variable Grouping

matrix partition (m_1, m_2, s_1, s_2) treat data as matrix $n = m_1 \times m_2$ and create $s_1 \times s_2$ blocks

diagonal extract main diagonals of $m \times m$ matrix

modulo partition

block partition

sliding window generator

triangular difference table

Generate Instances

- Combine Groups for generating ground parameters
  - individually
  - as pairs
  - as matrix
- Add arguments
  - as pattern
  - through functional dependency
  - avoid guessing

Relevance Check

- Constraint Program
  - For each group, a **variable** describes which constraint is used
- **Bi-criteria** optimization
  - Compactness of the conjunction generator
  - Ranking of the constraints in the conjunction

Compactness

- How **compact** is the selection of constraints
- Ideally, only one constraint used for all groups
- Or, regular pattern with **short period**
- Or, pattern with **few changes**

Ranking

- How likely is this constraint for these arguments
- Defined in detail for **Constraint Seeker** (*see seeker talk*)
- **Multi-criteria**
  - Argument structure (*functional dependency, crispness*)
  - Solution density (*approximation*)
  - Importance of constraint
  - Typical restrictions on constraint arguments
  - Implication between constraints

Dominance Check

- Certain conjunctions of constraints are **dominated** by others
- Weaker than full implication, syntactic check only
- Implications between constraints
- **Properties** of constraints arguments
  - **Contractible** (*alldifferent*)
  - **Extensible** (*atleast*)
- **New meta-data** in constraint catalog

Outline

1. Motivation
2. Approach
3. Evaluation

Magic Square of order n

- Take all numbers from 1 to n^2
- Arrange in $n \times n$ matrix
- All rows, columns and main diagonals must have the same sum

Famous Magic Square (Albrecht Duerer)

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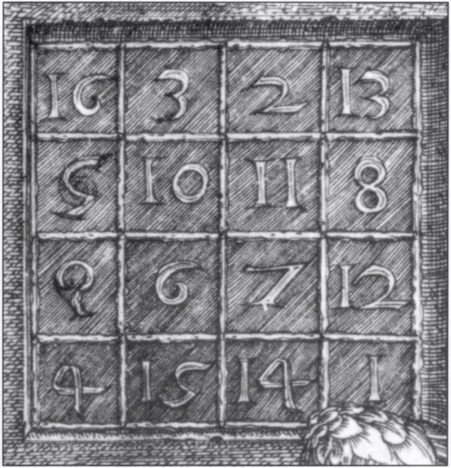
Input 16, 3, 2, 13, 5, 10, 11, 8, 9, 6, 7, 12, 4, 15, 14, 1



Generated Constraint Pattern (1)

Generator	matrix(16,1,16,1)
Partition	original sequence of values
Constraint(s)	1×alldifferent_consecutive_values 1×symmetric_alldifferent, extra parameter $[1, 2, \ldots, 16]$

What are these constraints?

alldifferent elements are pairwise different from each other

alldifferent_consecutive_values n elements are alldifferent and range from a to $a+n-1$

symmetric_alldifferent elements are alldifferent and $x_i = j \implies x_j = i$

Generated Constraint Pattern (2)

Generator	matrix(4,4,1,4)
Partition	16_1 3_2 2_3 13_4 / 5_5 10_6 11_7 8_8 / 9_9 6_{10} 7_{11} 12_{12} / 4_{13} 15_{14} 14_{15} 1_{16}
Constraint(s)	4×`sum_ctr`, extra parameters $=$, 34

Generated Constraint Pattern (3)

<table>
<tr><td>Generator</td><td colspan="4">matrix(4,4,4,1)</td></tr>
<tr><td rowspan="4">Partition</td><td>16_1</td><td>3_2</td><td>2_3</td><td>13_4</td></tr>
<tr><td>5_5</td><td>10_6</td><td>11_7</td><td>8_8</td></tr>
<tr><td>9_9</td><td>6_{10}</td><td>7_{11}</td><td>12_{12}</td></tr>
<tr><td>4_{13}</td><td>15_{14}</td><td>14_{15}</td><td>1_{16}</td></tr>
<tr><td>Constraint(s)</td><td colspan="4">$4\times$sum_ctr,
extra parameters $=$, 34</td></tr>
</table>

Generated Constraint Pattern (4)

Generator	matrix(8,2,4,1)
Partition	$\begin{array}{\|c\|c\|} \hline 16_1 & 3_2 \\ 2_3 & 13_4 \\ 5_5 & 10_6 \\ 11_7 & 8_8 \\ \hline 9_9 & 6_{10} \\ 7_{11} & 12_{12} \\ 4_{13} & 15_{14} \\ 14_{15} & 1_{16} \\ \hline \end{array}$
Constraint(s)	$4 \times$ `sum_ctr`, extra parameters $=$, 34

Generated Constraint Pattern (5)

Generator	matrix(2,8,2,2)
Partition	16_1 3_2 9_9 6_{10} | 2_3 13_4 7_{11} 12_{12} | 5_5 10_6 4_{13} 15_{14} | 11_7 8_8 14_{15} 1_{16}
Constraint(s)	$4\times$`sum_ctr`, extra parameters $=$, 34

Generated Constraint Pattern (6)

Generator	diagonal
Partition	16_1 3_2 2_3 13_4 5_5 10_6 11_7 8_8 9_9 6_{10} 7_{11} 12_{12} 4_{13} 15_{14} 14_{15} 1_{16}
Constraint(s)	2×sum_ctr, extra parameters =, 34 2×strictly_decreasing

Solutions to Generated Model

13	3	2	16
8	10	11	5
12	6	7	9
1	15	14	4

13	2	3	16
8	11	10	5
12	7	6	9
1	14	15	4

16	2	5	11
3	13	10	8
9	7	4	14
6	12	15	1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Can we learn basic model from random, positive samples?

- Select random subset of all solutions to 4x4 magic squares
- See how many constraint pattern are suggested
- Four constraint pattern required for basic magic square model
- Converges quite rapidly (*3 or 4 samples are enough*)

Balanced Incomplete Block Designs (v, b, r, k, λ)

- Consists of v distinct items and b blocks
- Each block contains k distinct objects
- Each item occurs in exactly r distinct blocks
- Two distinct items occur together in exactly λ blocks

Sample (7,7,3,3,1) Design

0	0	0	0	1	1	1
0	0	1	1	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	1	0	0	0	0	1

Constraint Pattern Found

Partition	Constraints
matrix(7,7,7,1)	all pairs: 21×scalar_product 7×sum_ctr, extra parameters =, 3 matrix: 1×lex_chain_less all pairs: 21×lex_less
matrix(7,7,1,7)	all pairs: 21×scalar_product 7×sum_ctr, extra parameters =, 3 matrix: 1×lex_chain_less all pairs: 21×lex_less
diagonal	2×no_peak

Orthogonal Latin Squares

Latin Square of order n A $n \times n$ matrix containing values 1 to n, such that each row and column contains each number from 1 to n exactly once

Orthogonal Latin Squares Two Latin Squares (a_{ij}) and (b_{ij}) are orthogonal, if the pairs $< a_{ij}, b_{ij} >$ are pairwise different

Example Orthogonal Latin Squares

0	2	1	3	4	6	5
6	1	4	2	5	3	0
1	0	6	5	3	4	2
2	5	0	4	6	1	3
5	3	2	6	1	0	4
3	4	5	1	0	2	6
4	6	3	0	2	5	1

0	6	5	2	1	4	3
5	0	4	3	2	1	6
2	1	3	5	0	6	4
1	4	2	0	6	3	5
6	3	0	1	4	5	2
4	5	1	6	3	2	0
3	2	6	4	5	0	1

data given as vector

0, 2, 1, 3, 4, 6, 5, 6, 1, 4, 2, 5, 3, 0, 1, 0, 6, 5, 3, 4, 2, 2, 5, 0, 4,
6, 1, 3, 5, 3, 2, 6, 1, 0, 4, 3, 4, 5, 1, 0, 2, 6, 4, 6, 3, 0, 2, 5, 1,
0, 6, 5, 2, 1, 4, 3, 5, 0, 4, 3, 2, 1, 6, 2, 1, 3, 5, 0, 6, 4, 1, 4, 2, 0,
6, 3, 5, 6, 3, 0, 1, 4, 5, 2, 4, 5, 1, 6, 3, 2, 0, 3, 2, 6, 4, 5, 0, 1

Constraint Pattern Found

Partition	Constraints
matrix(14,7,7,1)	14×`alldifferent_consecutive_values`
matrix(14,7,1,7)	14×`alldifferent_consecutive_values`
matrix(2,49,2,1)	1×`lex_alldifferent`
matrix(7,14,7,7)	2×`sum_ctr` with extra parameters =, 147
matrix(7,14,7,1) matrix(14,7,7,7) matrix(49,2,7,1) matrix(7,14,7,2) matrix(14,7,2,7) matrix(14,7,1,7)	1×`lex_alldifferent`

Points to Remember

- Structured conjunctions of similar global constraints seems to be the right degree of abstraction to concisely describe model for structured problems.
- Conjunction of similar global constraints are intelligible to the user.
- The structure restricts a lot and guides the search process.
- The whole approach takes advantage of meta data describing each constraint.
- Domination check is crucial for reducing number of candidates.

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