Weights in stable matching problems increase manipulation opportunities

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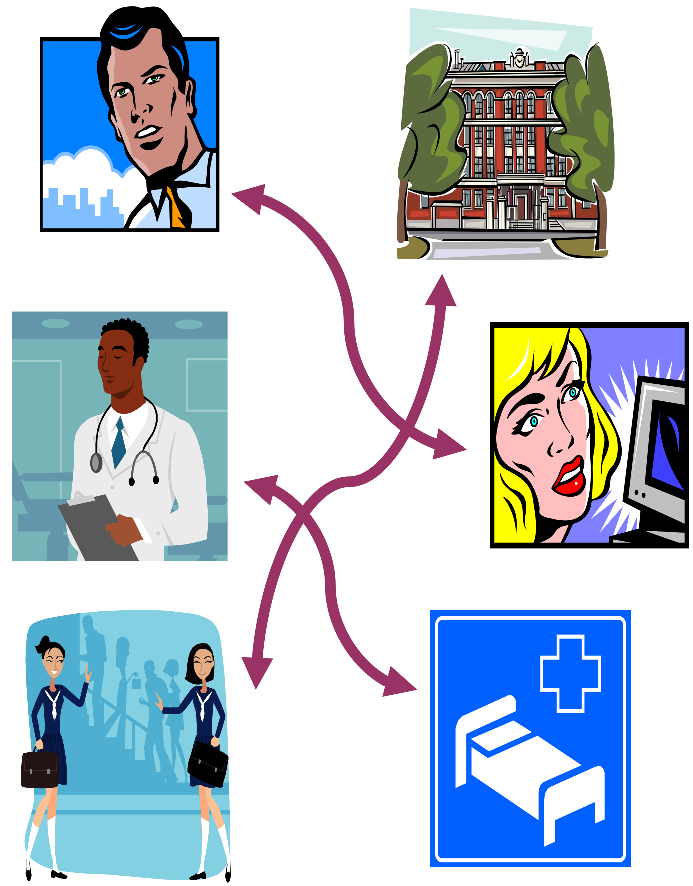
Stable marriage (SM)

- Matching of two sets
  - Men to Women
  - Doctors to Hospitals
  - Students to Schools …
- Preferences: strict total order over the members of the other set
- Stability: no man and woman, who are not married to each other, both prefer each other
- Gale-Shapley algorithm: Men are married to the best women possible (male optimal) and women are married to the worst men possible (female pessimal)



Stable marriage with weights (SMW)

- In some practical applications
  - It is more natural to express scores rather than a qualitative preference ordering
  - Scores model
    - quantitative preferences
    - profits or costs

- In SMWs
  - Each man provides a score for each woman
  - Each woman provides a score for each man

- When preferences are weighted
  - stability notions that rely on the scores
    - α-stability
    - link-stability

α-stability

- Definition
  - no man and woman, who are not married to each other, both prefer each other by at least a value α
- Generalization of the classical notion of stability for SMs

- Example

m_1: $w_1^{[3]} > w_2^{[2]}$
m_2: $w_1^{[4]} > w_2^{[1]}$
w_1: $m_1^{[8]} > m_2^{[5]}$
w_2: $m_1^{[4]} > m_2^{[1]}$

α-stable marriage when α=1
M_1: $\{(m_1, w_1), (m_2, w_2)\}$

α-stable marriages when α=2
M_1: $\{(m_1, w_1), (m_2, w_2)\}$
M_2: $\{(m_1, w_2), (m_2, w_1)\}$

Link-stability

- Link strength
  - The link strength of (m,w)

    $l(m,w)=s(m,w)+s(w,m)$
  - l(M): sum of the links of all pairs of M
  - Other possibilities instead of the sum: max, product
- Link-stability
  - No man and woman, who are not married to each other, both prefer each other in terms of their link
  - It is not a generalization of the classical notion of stability for SMs

- Example: SMW P

  m_1: $w_1^{[30]} > w_2^{[3]}$

  m_2: $w_1^{[4]} > w_2^{[3]}$

  w_1: $m_2^{[6]} > m_1^{[5]}$

  w_2: $m_1^{[10]} > m_2^{[2]}$

  link-stable marriage

  M_1: $\{(m_1, w_1), (m_2, w_2)\}$

  $l(M_1) = (30+5) + (3+2) = \mathbf{40}$

  Male-optimal stable marriage

  M_2: $\{(m_1, w_2), (m_2, w_1)\}$

  $l(M_2) = (3+10) + (4+6) = \mathbf{23}$

W-manipulation

- Every stable marriage procedure is manipulable (with at least 3 men and 3 women) [Roth 1982]
  - There is a profile where an agent, misreporting his preferences, obtains a better stable matching
- Two ways of manipulating
  - Changing the preference ordering
  - Truncating the preference lists
- In SMWs, another way of manipulating:
  - Changing the weights
  - W-manipulation: manipulation by just changing the weights (no truncation, no preference change)
  - We want to see if this gives additional manipulating power

W-manipulation

- A stable marriage procedure f is
  - w-manipulable if
    - There is a pair of profiles p, p’ that differ for the weights of one agent, say m, such that f(p’) is better than or equal to f(p) for m in p
  - Strictly w-manipulable if
    - There is a pair of profiles p, p’ that differ for the weights of one agent, say m, such that f(p’) is better than f(p) for m in p

W-manipulation for α-stability

- Suppose the manipulator knows the value of α
- Then:
  - Every procedure which returns an α-stable matching is w-manipulable
  - There is at least one procedure which is strictly w-manipulable

Example

- P

  m_1: $w_1^{[5]}$ > $w_2^{[3]}$

  m_2: $w_1^{[5]}$ > $w_2^{[3]}$

  w_1: $m_1^{[5]}$ > $m_2^{[4]}$

  w_2: $m_1^{[5]}$ > $m_2^{[3]}$

- P'

  m_1: $w_1^{[5]}$ > $w_2^{[3]}$

  m_2: $w_1^{[5]}$ > $w_2^{[3]}$

  w_1: $m_1^{[5]}$ > $m_2^{[3]}$

  w_2: $m_1^{[5]}$ > $m_2^{[3]}$

- α=2
- α-stable marriages in P:
  - $M_1 = \{(m_1,w_1), (m_2,w_2)\}$
  - $M_2 = \{(m_1,w_2), (m_2,w_1)\}$
- α-stable marriage in P': M_1
  - Better than M_2 for w_1
- Every procedure must return M_1 in P'
- Manipulation strategy: the manipulator eliminates a tie

Can we avoid it?

- Restrictions on the profiles
- No ties? It means eliminating the weights!
- At most one tie for each agent? Not useful (same example as before)
- At most one tie in whole profile? Same example as before
- So, if agents know the value of α, there is no way to prevent manipulation!
- The same holds also if agents only know that α is smaller than α_{max}

W-manipulation for link stability

- Thm: Every procedure that returns a link stable matching is strictly w-manipulable
- Link stable marriages in P
  - $M_1 = \{(m_1,w_2), (m_2,w_1)\}$
- Link stable marriage in P'
  - $M_2 = \{(m_1,w_1), (m_2,w_2)\}$
  - Better than M_1 for w_1 in P
- Only one stable matching in P and P', so every procedure will return it

- P

m_1: $w_2^{[6]} > w_1^{[4]}$

m_2: $w_2^{[5]} > w_1^{[4]}$

w_1: $m_1^{[4]} > m_2^{[3]}$

w_2: $m_1^{[3]} > m_2^{[2]}$

- P'

m_1: $w_2^{[6]} > w_1^{[4]}$

m_2: $w_2^{[5]} > w_1^{[4]}$

w_1: $m_1^{[5000]} > m_2^{[2]}$

w_2: $m_1^{[3]} > m_2^{[2]}$

Manipulation strategy: the manipulator sets a very high weight for his top choice

Profile restrictions

- Possible way to avoid this form of manipulation: force the same weight for all top choices
- Thm: if same weight for all top choices every procedure is w-manipulable, and there is at least one which is strictly w-manipulable
- If same weight for all top choices and all differences equal to 1, then fixed weights (and thus irrelevant)
- At most one difference =2, and all others =1, same Thm.

Conclusions

- For α-stability adding weights increases the possibility of manipulating
  - Manipulation is possible by just changing the weights (no preference changing, nor list truncation)
  - Reasonable restrictions over the weights do not help
- For link stability, forcing same weight for all top choices prevents dictatorship of the manipulator

Thank you!

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