Weights in stable marriage problems increase manipulation opportunities

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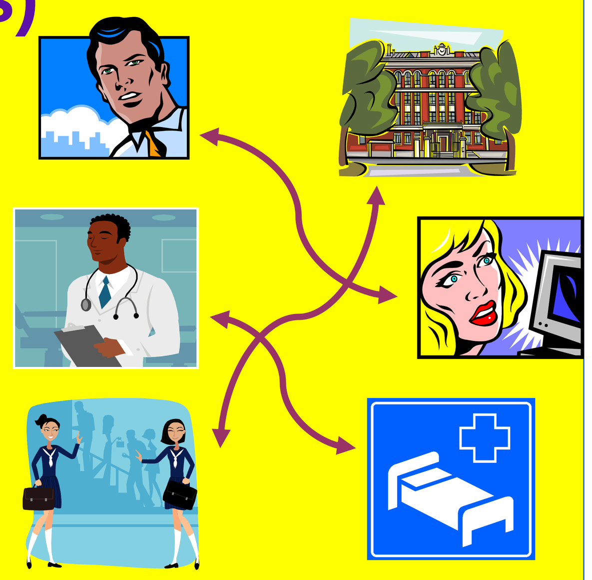
Is it possible to manipulate by just modifying the weights?

Stable marriage problems (SMs)

Matching elements of two sets

- Men to Women
- Doctors to Hospitals
- Students to Schools ...



Preferences: strict total order over the members of the other set

Stability: no man and woman, who are not married to each other, both prefer each other

A stable marriage always exists

Gale Shapley algorithm: Men are married to the best women possible and women are married to the worst men possible

Every stable marriage procedure is manipulable

- There is a profile where an agent, misreporting his preferences, obtains a better stable matching
- Two ways of manipulating:
  - → Changing the preference ordering
  - → Truncating the preference lists

Stable marriage problems with weights (SMWs)

Preferences: a score for each member of the other set

α-stability: no man and woman, who are not married to each other, both prefer each other by at least α

link-stability: no man and woman, who are not married to each other, both prefer each other in terms of their link

link(m,w) = f(score(m,w),score(w,m)) f=sum,max,min,…

W-manipulation in SMWs

Another way of manipulating:

→ Just changing the weights (no truncation, no preference change)

A stable marriage procedure is

- **w-manipulable** if ∃ profiles p, p' that differ for the weights of one agent, say m, such that f(p') is better than or equal to f(p) for m
- **Strictly w-manipulable:** f(p') is better than f(p) for m in p

W-manipulation for α-stability

The manipulator knows the value of α

- Every procedure is w-manipulable
- At least one procedure is strictly w-manipulable

Example (α=2)

P
$m_1: w_1^{[5]} > w_2^{[3]}$
$m_2: w_1^{[5]} > w_2^{[3]}$
$w_1: m_1^{[5]} > m_2^{[4]}$
$w_2: m_1^{[5]} > m_2^{[3]}$

P'
$m_1: w_1^{[5]} > w_2^{[3]}$
$m_2: w_1^{[5]} > w_2^{[3]}$
$w_1: m_1^{[5]} > m_2^{[3]}$
$w_2: m_1^{[5]} > m_2^{[3]}$

α-stable in P: $M_1 = \{(m_1,w_1), (m_2,w_2)\}$
$M_2 = \{(m_1,w_2), (m_2,w_1)\}$
α-stable in P': only M_1 (better than M_2 for w_1 in P)
Every procedure must return M_1 in P'

Can we avoid this form of manipulation?

- Restrictions on the profiles
- No ties? It means eliminating the weights!
- At most one tie for each agent? Not useful (same example as before)
- At most one tie in whole profile? Same example as before

So, if agents know the value of α, there is no way to prevent manipulation!
The same holds also when the agents know a lower bound for α, or know nothing about α

W-manipulation for link-stability

- Assume f=sum
- Every procedure is strictly w-manipulable

Example

P
$m_1: w_2^{[6]} > w_1^{[4]}$
$m_2: w_2^{[5]} > w_1^{[4]}$
$w_1: m_1^{[4]} > m_2^{[3]}$
$w_2: m_1^{[3]} > m_2^{[2]}$

P'
$m_1: w_2^{[6]} > w_1^{[4]}$
$m_2: w_2^{[5]} > w_1^{[4]}$
$w_1: m_1^{[500]} > m_2^{[2]}$
$w_2: m_1^{[3]} > m_2^{[2]}$

link-stable in P: $M_1 = \{(m_1,w_2), (m_2,w_1)\}$
link-stable in P': M_2 (better than M_1 for w_1 in P)
Every procedure must return M_2 in P'

Can we avoid this form of manipulation?

- In the example above w_1 sets a very high weight for her top choice → surely matched to such a top choice!
- Possible way to avoid it: force the same weight for all top choices
- Thm.: **If same weight for all top choices, every procedure is w-manipulable, and there is at least one which is strictly w-manipulable**
- If same weight for all top choices and all differences equal to 1, then fixed weight (and thus irrelevant)
- Same result when at most one difference =2, and all others =1