Maria Silvia Pini
Francesca Rossi
K. Brent Venable

University of Padova, Italy

COMPUTATIONAL SOCIAL CHOICE

General information

- When and where:
  - 20 Giugno: 11-13 e 14-16, aula 1BC50
  - 21 Giugno: 11-13 e 14-16, aula 2AB40
  - 26 Giugno: 11-13 aula 2AB45, 14-16 aula1BC50
  - 27 Giugno: 11-13 e 14-16, aula 2AB45
  - 28 Giugno: 11-13 e 14-16, aula 1BC45
- Who:
  - Maria Silvia Pini, pini@dei.unipd.it
  - Francesca Rossi, frossi@math.unipd.it
  - K. Brent Venable, kvenable@math.unipd.it

Outline

1. Introduction, Motivation and Overview (Venable)
2. Voting theory: procedures and properties (Venable)
3. Characterization and Impossibility theorems (Venable)
4. Computational aspects of social choice (Pini)
5. Uncertainty in preference aggregation (Pini)
6. Compact preference representation (Rossi)
7. Matching Problems (Rossi)

What are we going to talk about

Collective decision making
Economics, Political Sciences

SOCIAL CHOICE THEORY ⟷ COMPUTER SCIENCE

Social welfare, Fairness…. → Societies of artificial agents

Voting procedures,
Fair division algorithms ← Complexity analysis,
algorithm design…

In more detail

- Social Choice gives us the problem e.g.:
  - electing a winner given individual preferences over candidates
  - aggregating individual judgments into a collective verdict
  - fairly dividing a cake given individual tastes

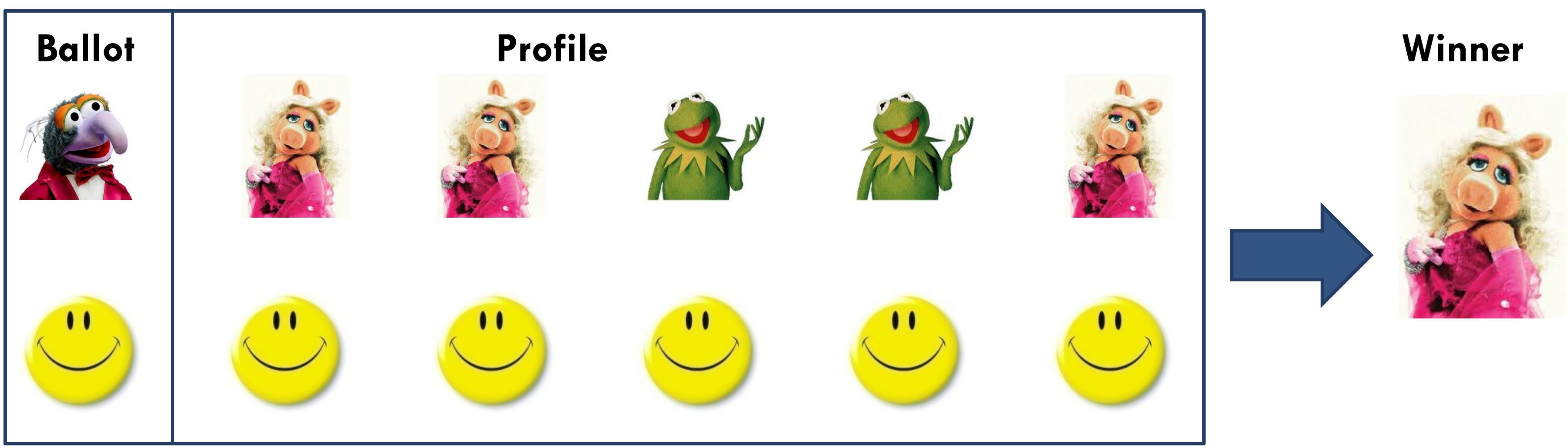
- We provide the computational technique, e.g:
  - algorithm design to implement complex mechanisms
  - complexity theory to understand limitations
  - logical modelling to fully formalise intuitions
  - knowledge representation techniques to compactly model problems
  - deployment in a multiagent system

Applications

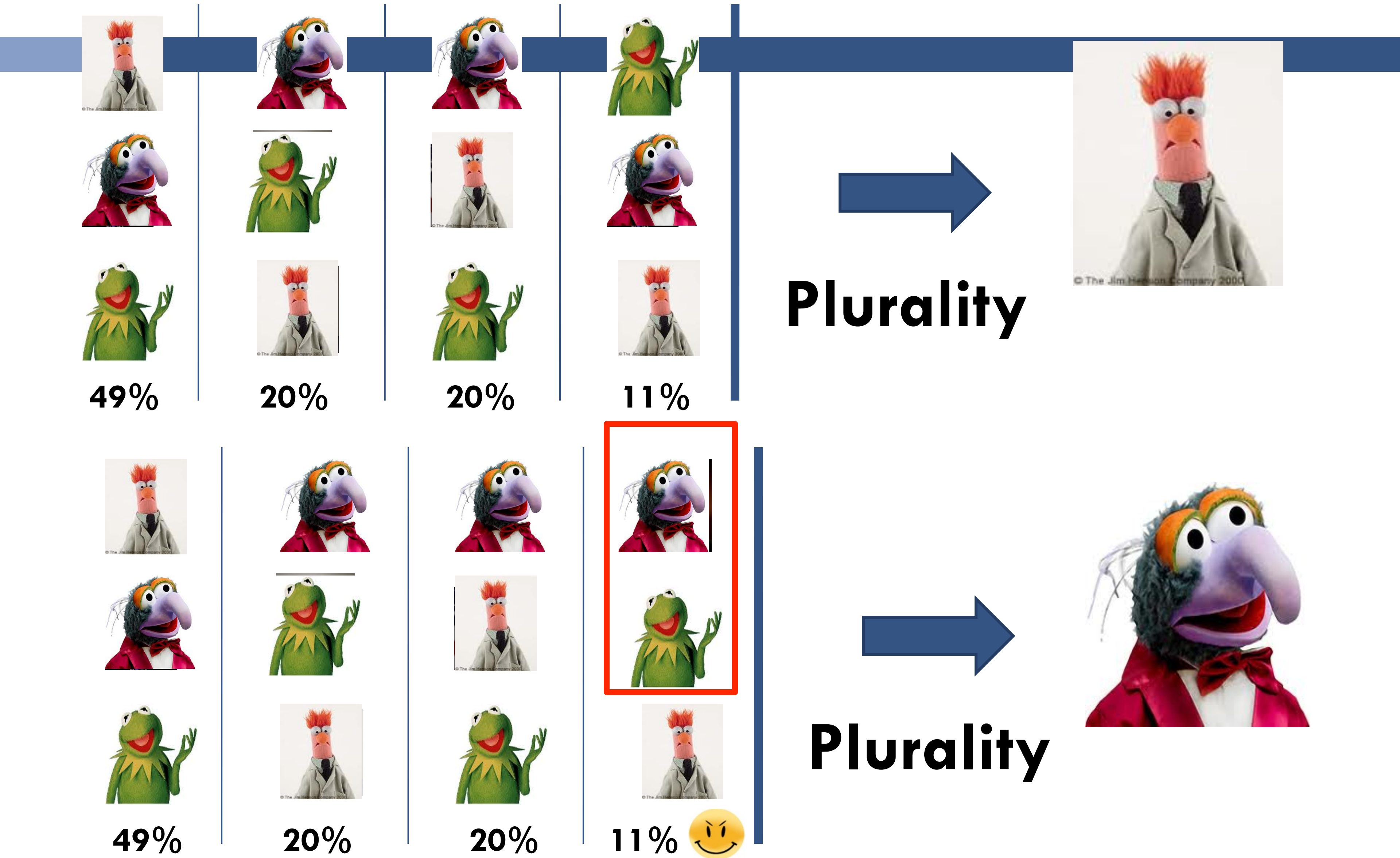
- Meta search engine
- Importance of a web page
- Sensor fusion
- Collaborative filtering in recommender systems
- Ontology merging in the Semantic Web

Plurality

- Ballot: 1 alternative
- Result: alternative(s) with the most vote(s)
- Example:
  - 6 voters
  - Candidates:



Could someone be better off lying?



Complexity of Manipulation

- **TH: Manipulability(Plurality)∈** P
- Proof
- Simply vote for x, the alternative to be made winner by means of manipulation. If manipulation is possible at all, this will work. Otherwise not.

[Bartholdi,Tovey,Trick,1989]

Uncertainty in preference aggregation

Example:

- 6 voters
- Candidates: 

Profile

Plurality

Possible Winners

Profile

Plurality

Necessary Winner

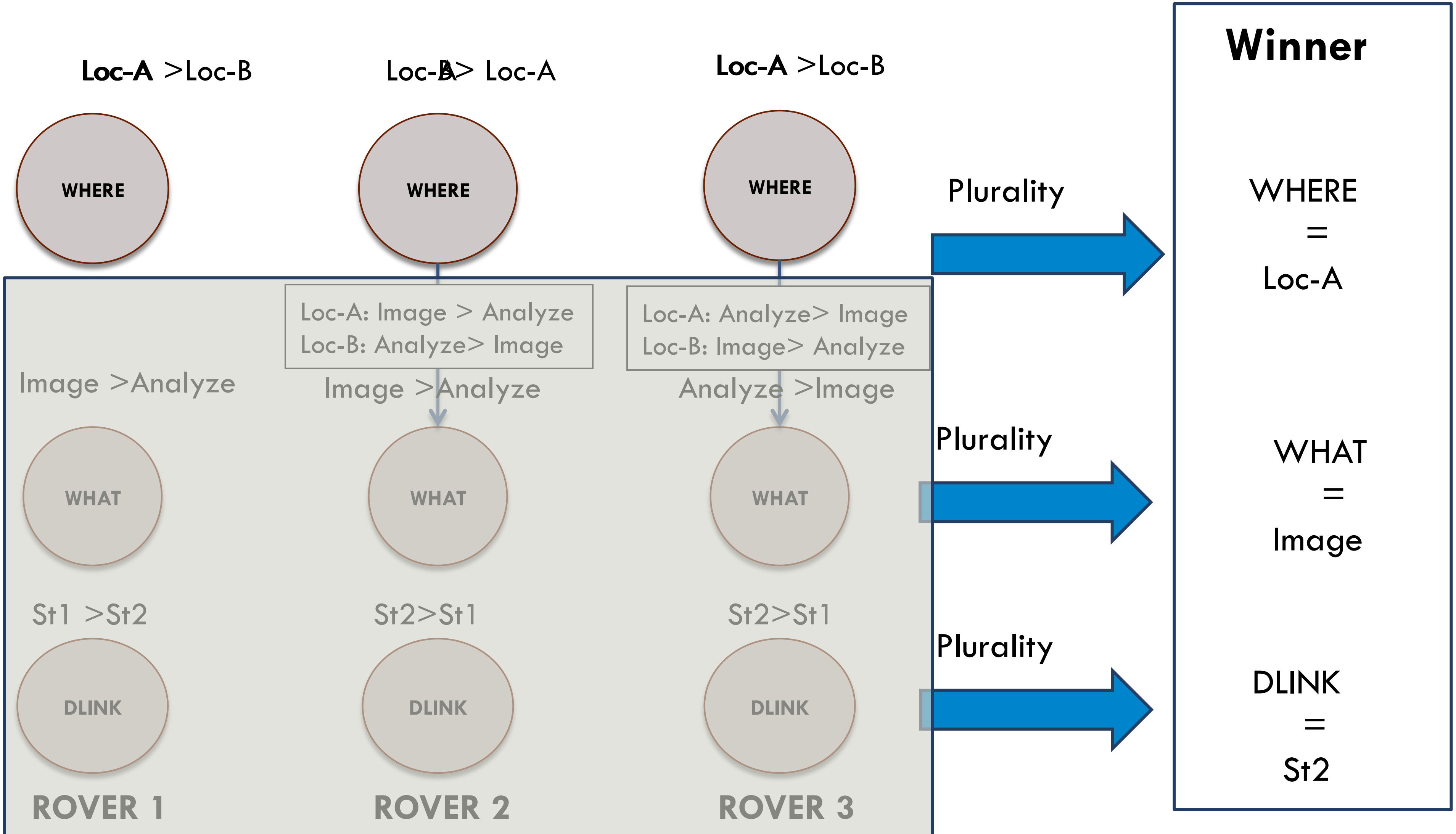
Compactness → combinatorial structure for the set of decisions

- Example:
  - Three friends need to decide what to cook for dinner
  - 4 items (pasta, main, dessert, drink)
  - 5 options for each → $5^4 = 625$ possible dinners
- In general: Cartesian product of several variable domains
- A compact representation of the preferences is needed

Voting with compact preferences

3 Rovers must decide:

- Where to go: Location A or Location B
- What to do: Analyze a rock or Take and image
- Which station to downlink the data to: Station 1 or Station 2



Matching Problems

- The rovers have decided to go at Loc-A, and they have to perform an analysis
  - One drills
  - One takes pictures
  - One downlinks data
- Two sets:
  - {Rover1, Rover2, Rover3}
  - {Drill,Picture,Download}
- Goal
  - find a stable matching

- Rovers
  - Rover1: downlink>picture>drill
  - Rover2: downlink>picture>drill
  - Rover3: downlink>picture>drill
- Tasks (e.g. mission coordinator)
  - Drill: Rover1>Rover2>Rover3
  - Picture: Rover2>Rover3>Rover1
  - Downlink: Rover3>Rover1>Rover2

Voting Theory

Voting procedures
Choice theoretic properties
Characterization Theorems
Impossibility and Possibility Theorems

Voting Procedures

- **n voters** (individuals, agents, players)
- **m candidates** (or alternatives)
- goal: collective choice among the candidates
- Each voter gives a **ballot**
  - the name of a single alternative,
  - a ranking (=linear orders of all alternatives …
- **Profile**: a set of n ballots (one for each voter)

Voting Procedures

- The procedure defines
  - the valid ballots
  - how they are aggregated
- Different types of result
  - Resolute voting procedures: **a single winner**
  - Voting correspondences: **a set of winners**
  - Social welfare functions: **an ordering over the set of candidates**

Resoluteness and Tie-breaking

- More formally
  - X: set of candidates
  - N: set of voters
  - L(X): set of linear orders over X
- (Resolute) Voting rule $F: L(X)^N \rightarrow X$
- (Irresolute) Voting correspondence $C: L(X)^N \rightarrow 2^X$
- Tie breaking rule: $T: 2^X - \{\} \rightarrow X$

- The composition of a voting correspondence with a tie breaking rule is a resolute voting rule

Overview of voting rules

- Positional Scoring Rules, e.g.:
  - Plurality
  - Borda
  - Veto
  - k-approval
- Plurality with Runoff
- Single Transferable Vote (STV)
- Approval Voting
- Condorcet-consistent methods based on the simple majority graph, e.g.:
  - Cup Rule/Voting Trees
  - Copeland
  - Banks
  - Slater
  - Schwartz,
  - Condorcet rule
- Condorcet-consistent methods based on the weighted majority graph, e.g:
  - Maximin/Simpson
  - Kemeny
  - Ranked Pairs/Tideman
- Condorcet-consistent methods requiring full ballot information, e.g.:
  - Bucklin
  - Dodgson
  - Young
- Majoritarian Judgment;
- Cumulative Voting;
- Range Voting.

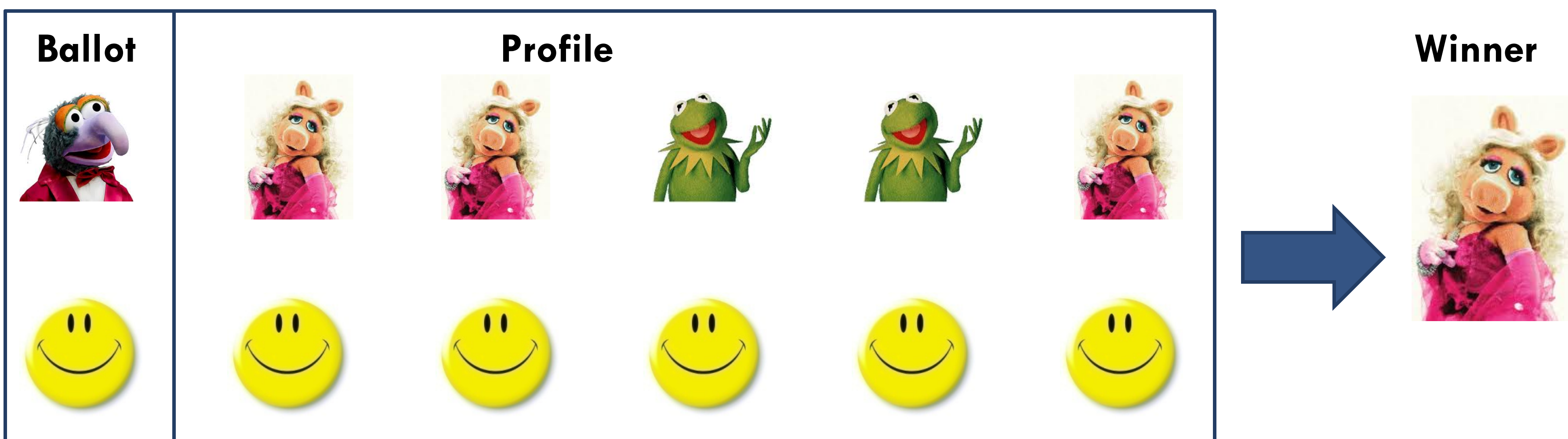
Positional scoring rules

Positional scoring rule

- Each candidate gets points for being ranked in a certain position by a voter
- Candidate score: sum of its points
- Winner: candidate(s) with the highest number of points

Plurality(1)

- Ballot: 1 alternative
- Result: alternative(s) with the most vote(s)
- Example:
  - 6 voters
  - Candidates:



Plurality(2)

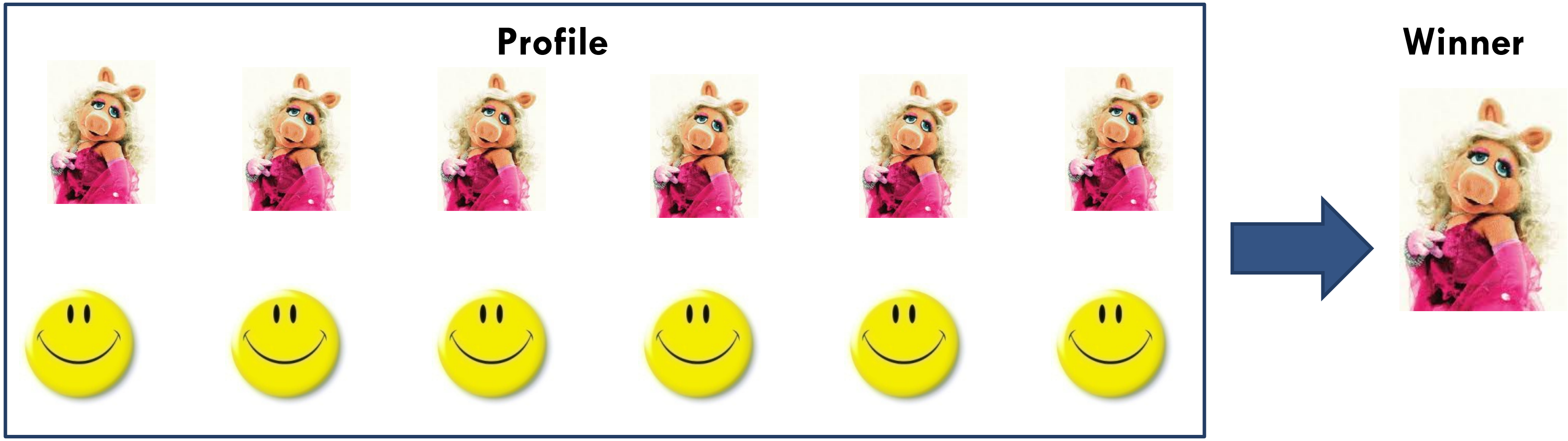
- Also called **simple majority** ($\neq$ absolute majority)
- Most widely used voting procedure
- If there are only two alternatives it is the best possible procedure (May's theorem)
- In any race with more than two candidates, plurality voting may elect the candidate least acceptable to the majority of voters.
- The **information** on voter preferences other than who their favorite candidate is gets **ignored.**
- **Dispersion** of votes across ideologically **similar candidates.**
- Encourages voters not to vote for their true favorite, if that candidate is perceived to have little chance of winning

Unanimity and Pareto Condition

- A voting procedure is **unanimous** if it elects (only) x whenever all voters say that x is the best alternative.

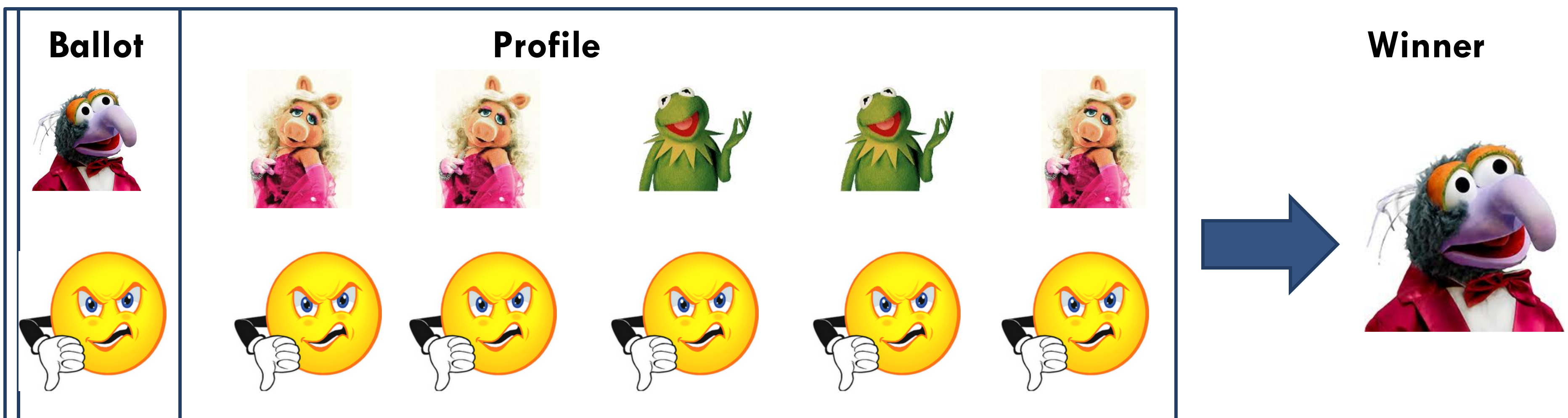
- The **weak Pareto condition** holds if an alternative y that is dominated by some other alternative x in all ballots cannot win.
- Pareto condition entails unanimity, but the converse is not true.

Plurality satisfies unanimity



Veto

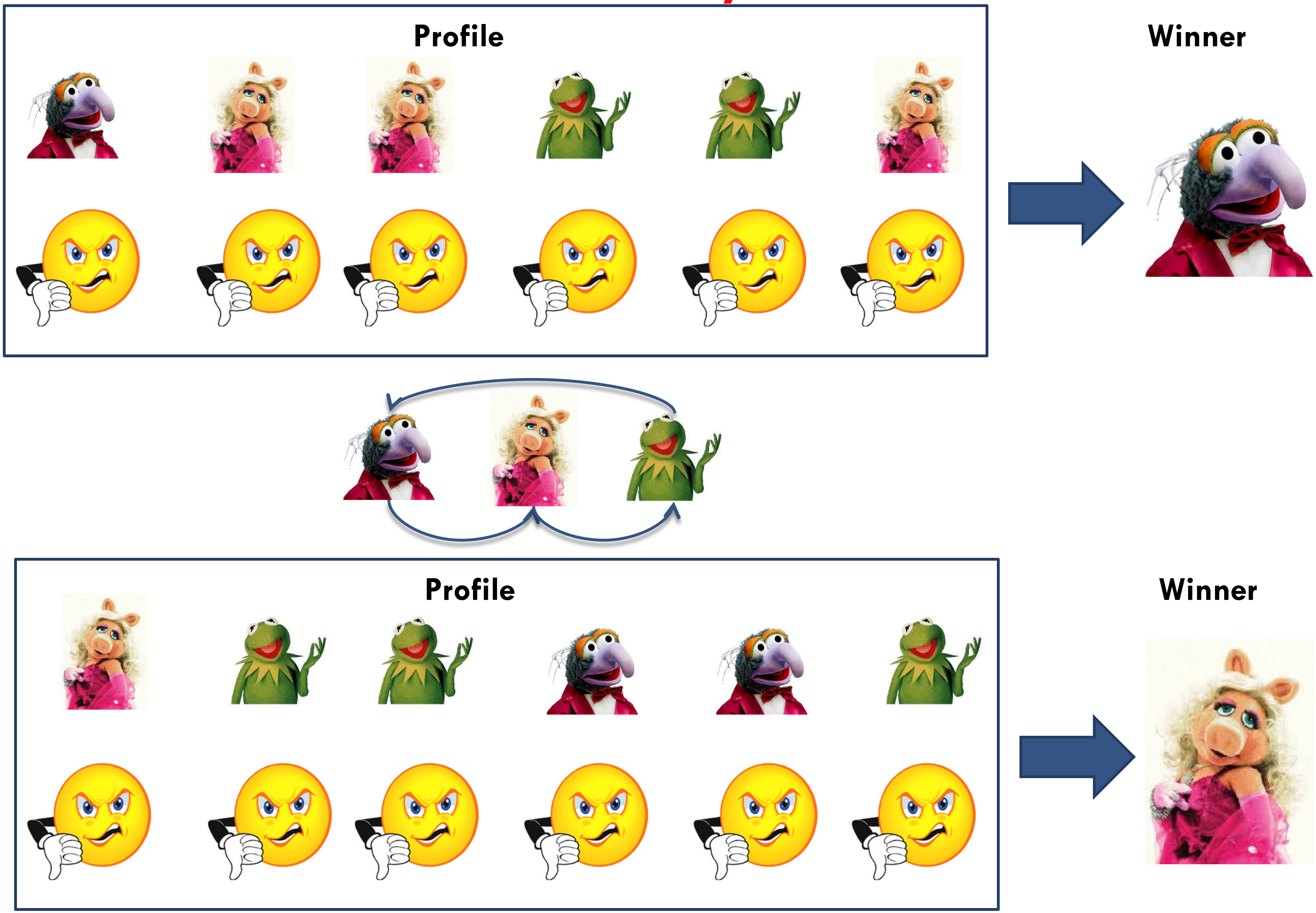
- Ballot: 1 vetoed alternative
- Result: candidate with the least vetos
- Example:
  - 6 voters
  - Candidates:



Neutrality

- If the names of the alternatives are permuted in the preferences of the voters, then the alternative selected by the voting rule change accordingly.

Veto satisfies neutrality

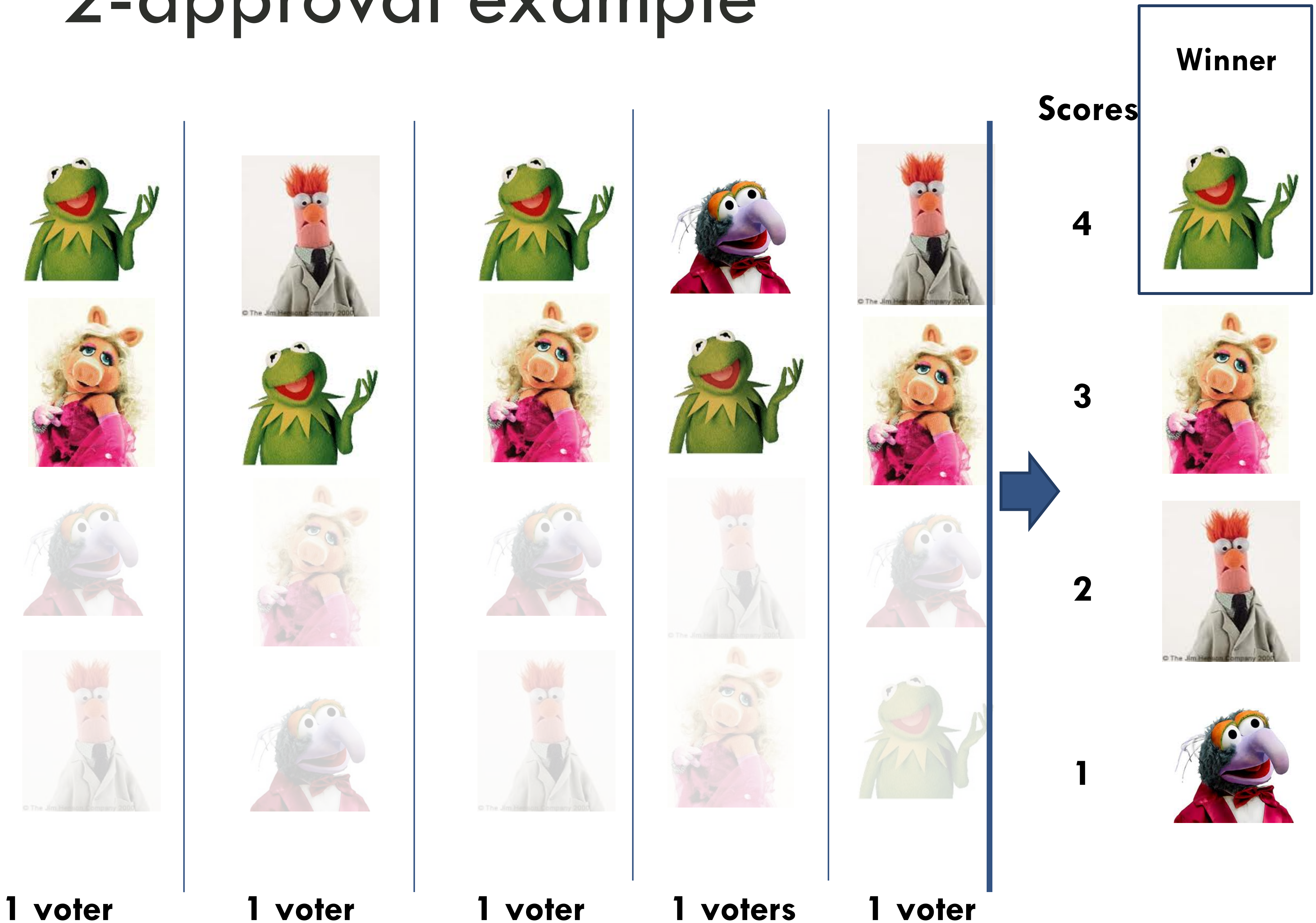


k-Approval

- **Ballot:** k favorite candidates
- **Procedure:**
  - for each voter
    - Each approved candidate gets one point
  - The score is the sum of all the points. The candidate(s) with the highest score win.
  - May need to tie break

- More informative balloting

2-approval example



Anonymity

- A voting rule is **anonymous** if the voters are treated symmetrically: if two voters switch ballots, then the winners don't change.

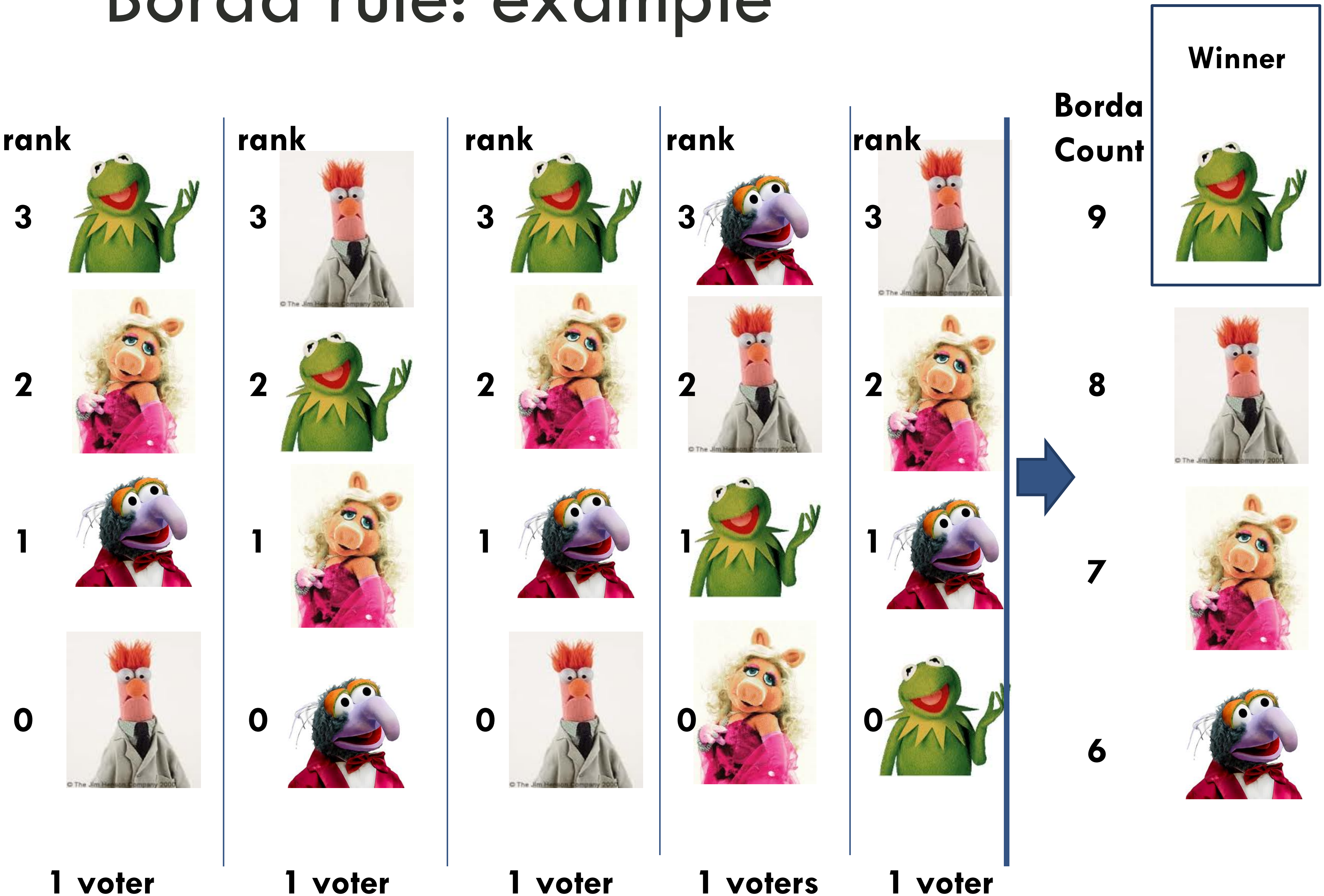
K-approval satisfies anonymity

Borda rule

- **Ballot:** complete ranking of all m candidates
- **Procedure:**
  - for each voter
    - candidate ranked 1st gets $m-1$ points
    - candidate ranked 2nd gets $m-2$ points
    - …
  - Borda count is the sum of all the points. The candidates with highest Borda count win.
- Proposed by Jean-Charles de Borda

- More informative balloting
- Higher elicitation and communication costs

Borda rule: example

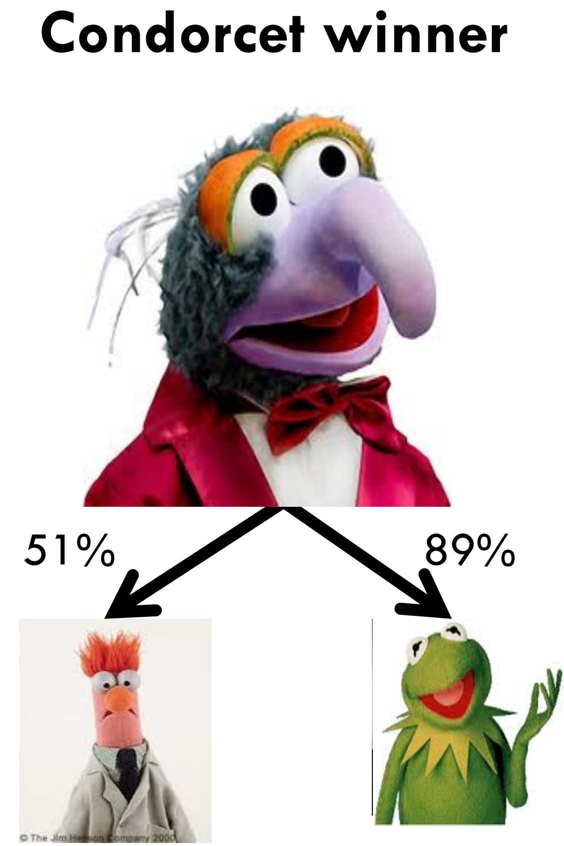


Positional scoring rule

- **Ballot**: complete ranking of all *m* candidates
- **Procedure**
  - Scoring vector $<s_1,s_2,\ldots s_m>$
  - s_i = points the candidate gets for being in position i for a voter
  - Count is the sum of all the points. The candidates with the highest count win.
- Examples of scoring vectors
  - Plurality: <1,0,…,0>
  - Veto: <1,1,…,1,0>
  - K-approval $<\underbrace{1,1,\ldots,1}_{k},0,0,\ldots,0>$
  - Borda: <m-1,m-2, …,1,0>

Condorcet Principle

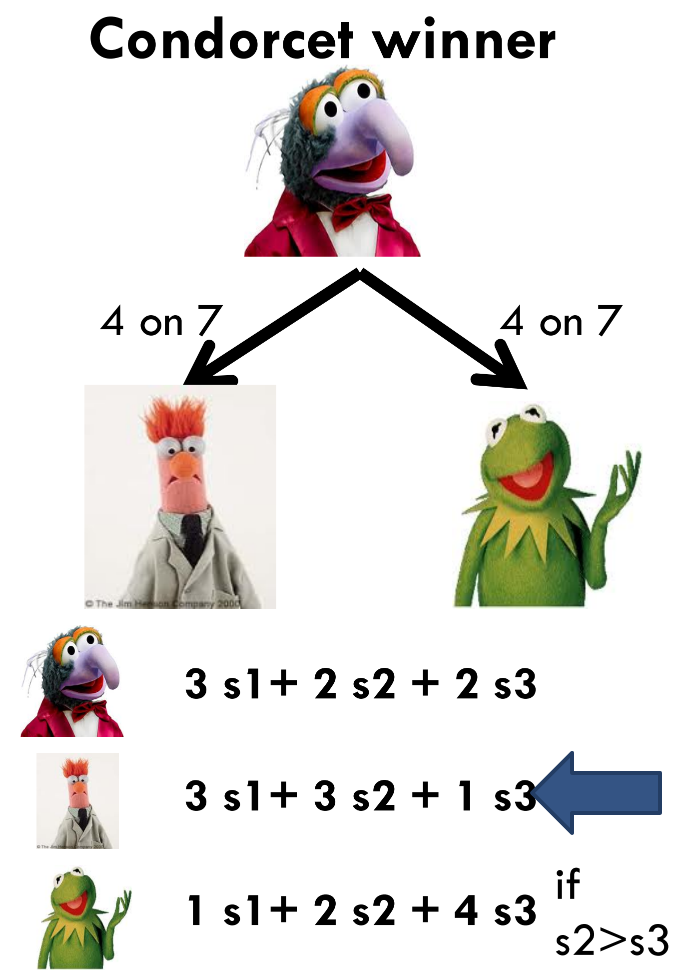
- Condorcet winner: an alternative that beats every other alternative in pairwise majority contests (if exists, unique)



Condorcet Consistency

- A voting rule is Condorcet consistent if, whenever there is a Condorcet winner, it is returned as the winner

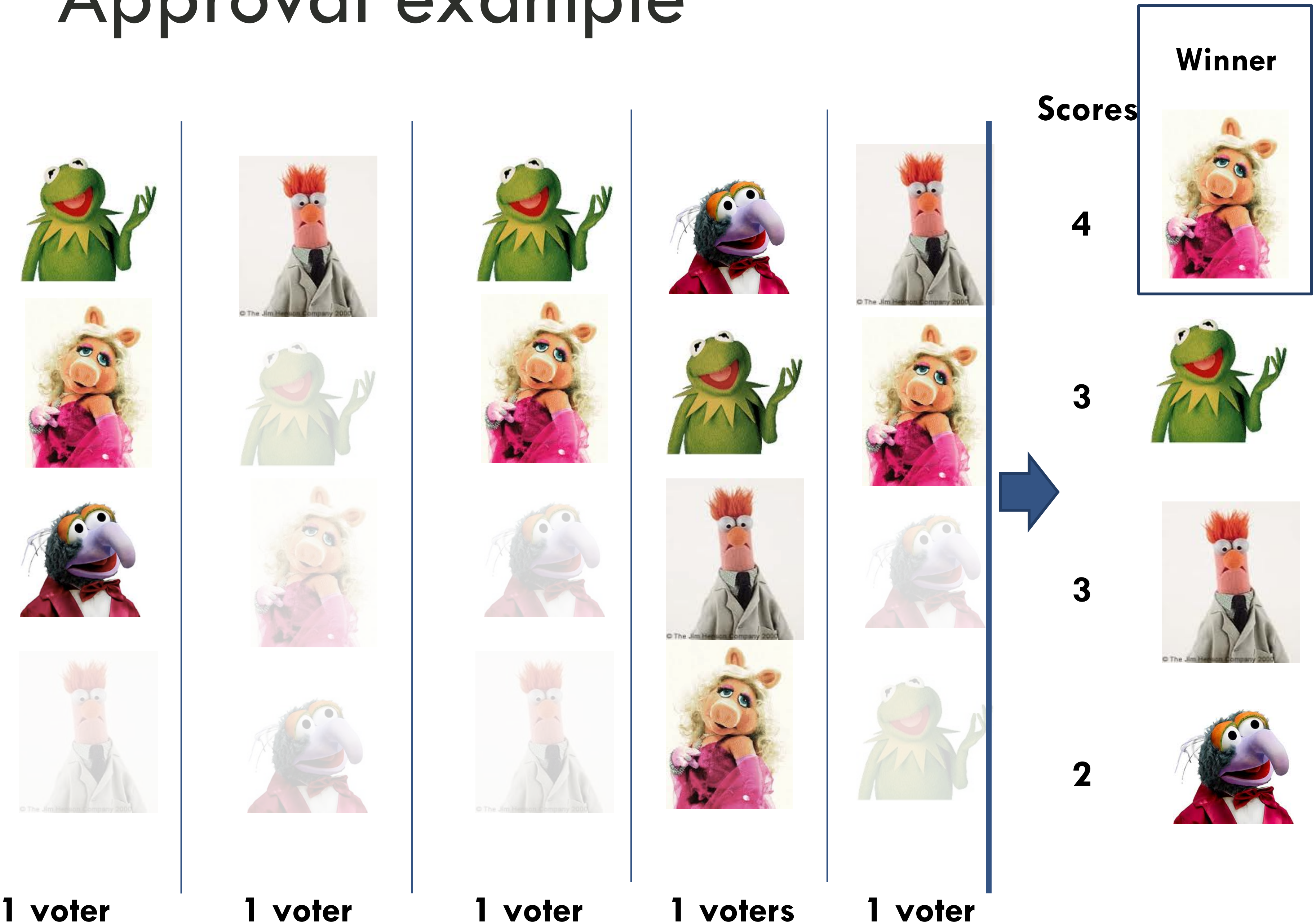
Positional scoring rules are not Condorcet Consistent



Approval

- **Ballot:** a set of favorite candidates
- **Procedure:**
  - for each voter
    - Each approved candidate gets one point
  - The score is the sum of all the points. The candidates with the highest score win.
  - May need to tie break
  - Named so by Weber in 1977
  - Widely used
  - Allows to express very different preferences

Approval example



Approval voting(2)

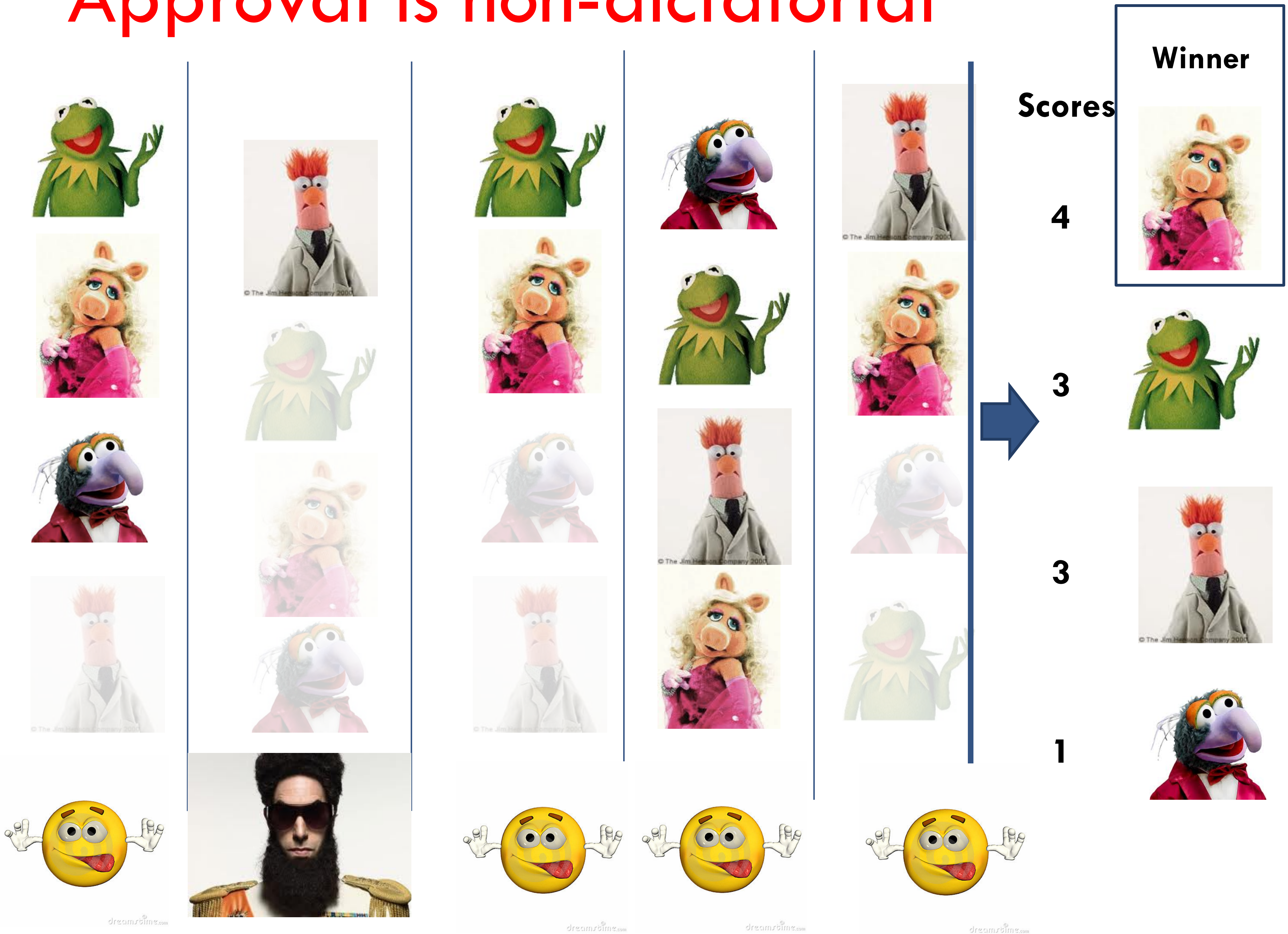
- Allows voters to vote for as many candidates as they find acceptable. For instance, a minor-party favorite an acceptable major-party candidate.
- There is no ranking; the candidate with the most approval votes wins, ensuring that the winning candidate is acceptable to the largest fraction of the electorate.
- Reduce negative campaigning, encouraging candidates to make more positive appeals to gain support from voters with primary commitments to other candidates.

- Can result in the defeat of a candidate who would win an absolute majority in a plurality system
- Can allow a candidate to win who might not win any support in a plurality elections,
- Has incentives for tactical voting

Dictatorship

- A voting procedure is dictatorial if there exists a voter (the dictator) such that the unique winner will always be his top-ranked alternative.
- A voting procedure is **non-dictatorial** if it is not dictatorial.
- Any anonymous voting procedure is non-dictatorial

Approval is non-dictatorial



Plurality with runoff(1)

- **Ballot:** 1 alternative
- **Procedure:** 2 rounds
  - 1st round: the top two choices are selected
  - 2nd round: plurality on the top two choices
- Example:
  - 5 voters
  - Candidates:

1st round

Winner

Plurality with runoff (2)

- Used to elect the president in France
- Elicits more information from voters: second best gets another chance
- Solves some problems of plurality:
  - Winner without a majority
  - Spoiler candidates
- Does not solve vote splitting
  - candidato least preferred by a majority may win

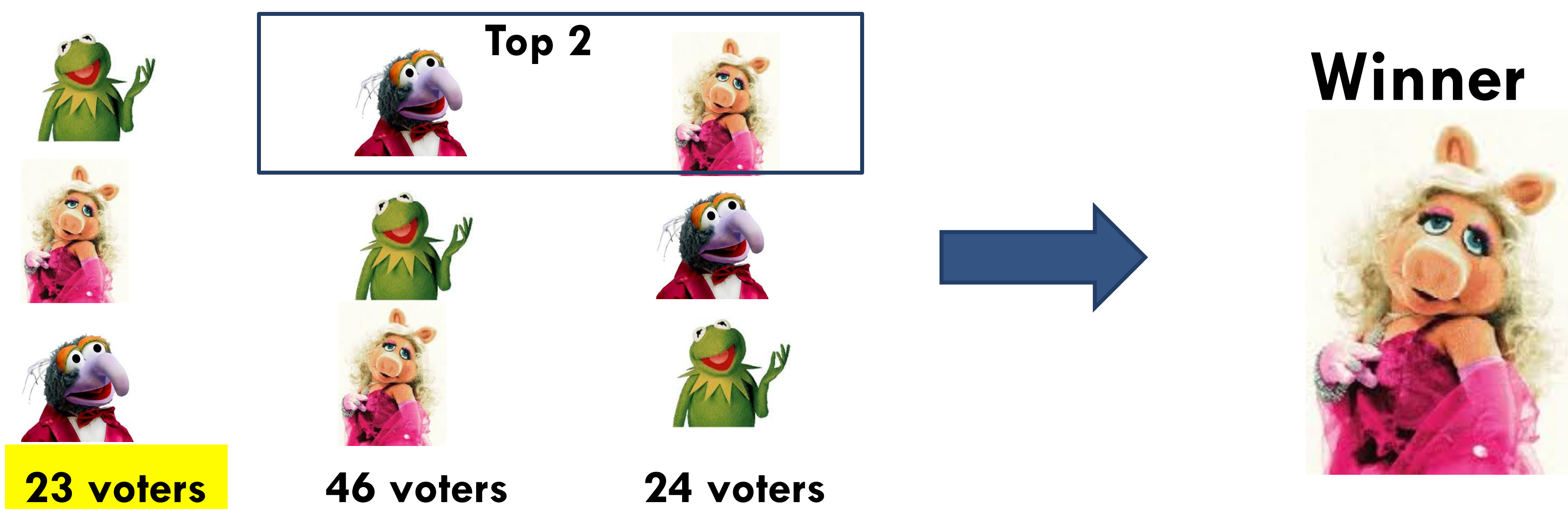
- Still: heavily criticized when Le Pen entered run-off in 2002

Participation

- Given a voter, his addition to a profile leads to an equally or more preferred result for this voter
- No incentive to abstain

Plurality with run-off is not participative

- With plurality with run-off it may be better to abstain than to vote for your favorite candidate!



Single Transferrable Vote (STV)

- **Ballot:** ranking of candidates
- **Procedure:**
  - If one of the candidates is the 1st choice for over 50% of the voters (quota), she wins.
  - Otherwise, the candidate who is ranked 1st by the fewest voters gets eliminated from the race.
  - Votes for eliminated candidates get transferred: delete removed candidates from ballots and "shift" rankings (i.e., if your 1st choice got eliminated, then your 2nd choice becomes 1st).
- Used in Australia, New Zeland etc.

STV: example

- At least 4 candidates otherwise is like Plur. with run-off

Single Transferrable Vote (2)

- Minimizes the number of wasted votes
- Before computers it was criticized for its complexity
- Allows the transfer of votes to a candidate from voters of another party → mitigates partisanship
- Interesting in terms of complexity of manipulation

Majority-graph-based rules

Based on pair-wise competitions between candidates

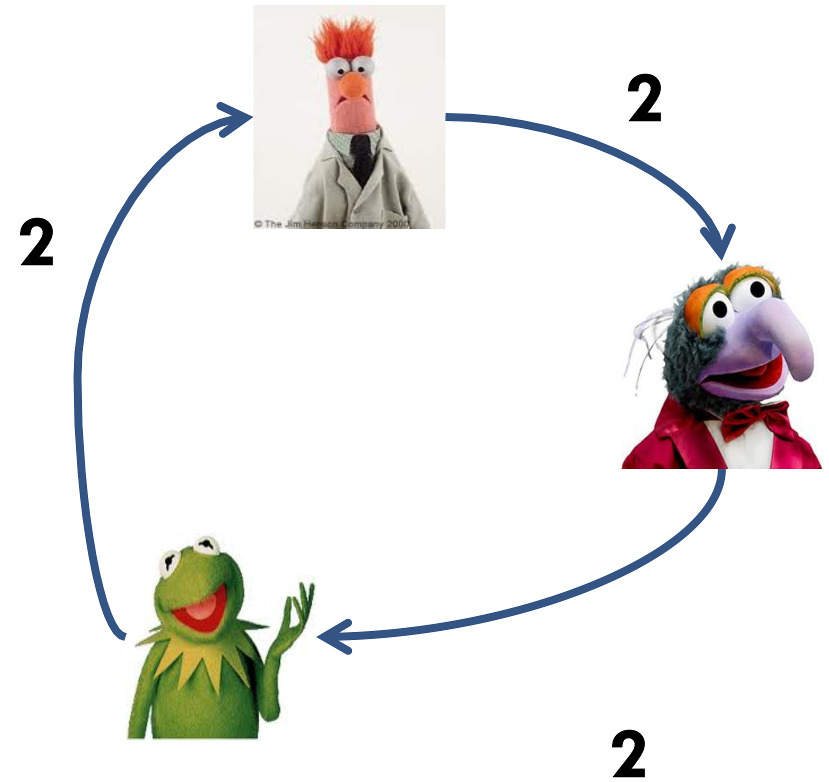
All Condorcet-consistent

Different choice when there is no Condorcet winner

Condorcet Paradox

- There may be no Condorcet winner

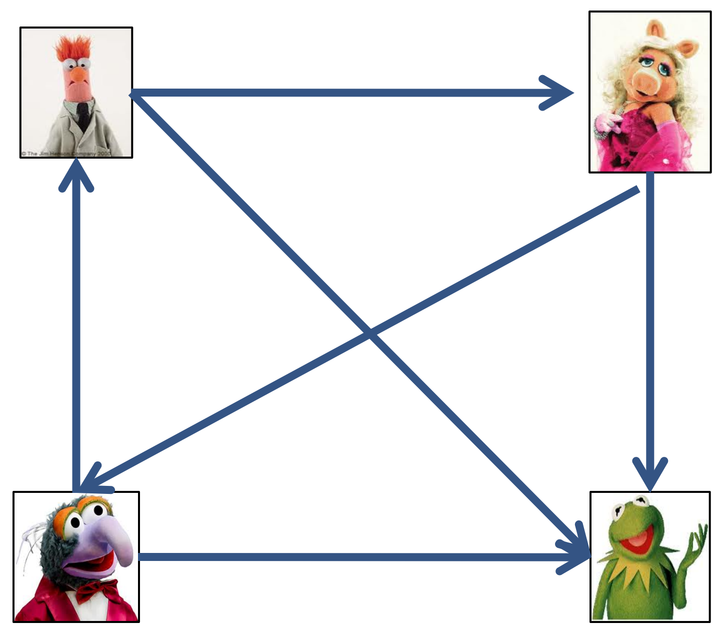
1 voter | 1 voter | 1 voter



Majority Graph

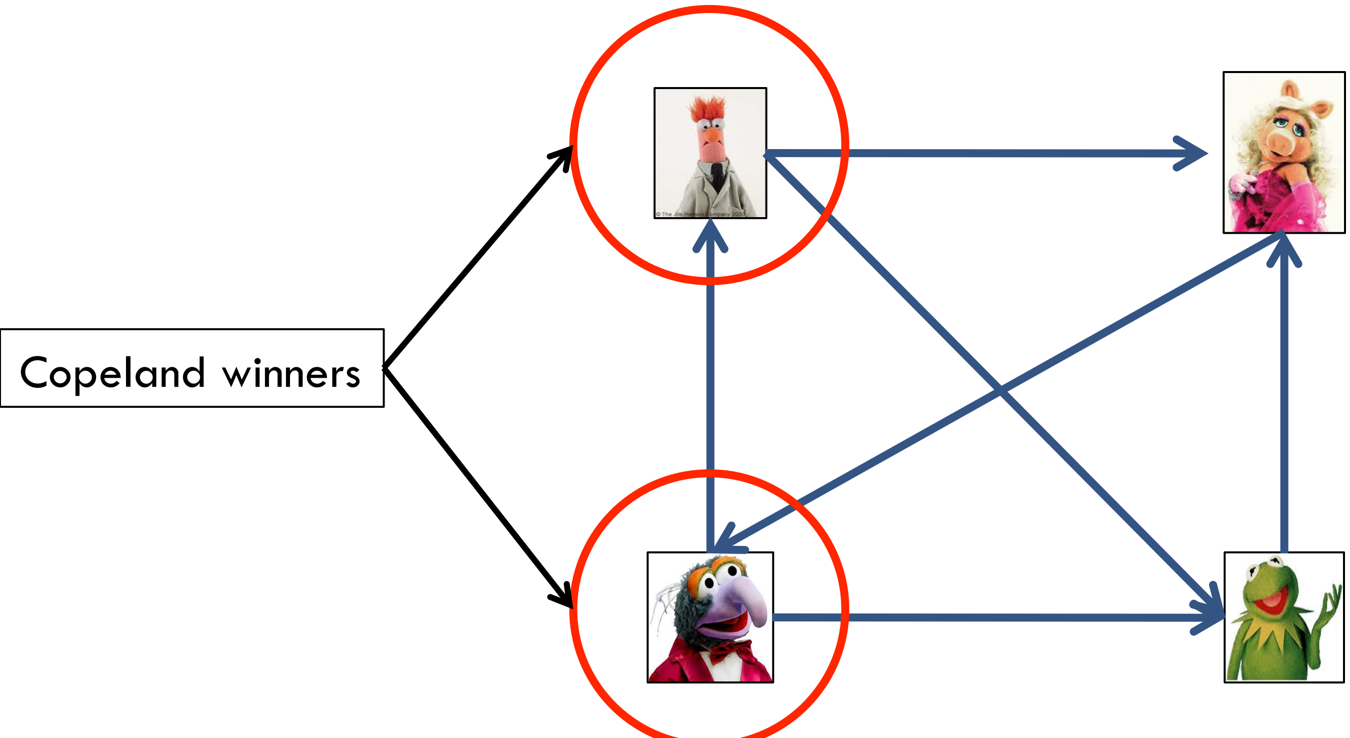
- **Ballot:** complete ranking of candidates
- **Majority graph**
  - One **node** for each **candidate**
  - **A→B** iff a **majority** of voters prefer **A over B**
  - In general not transitive (Condorcet paradox)
  - May be weighted

2 voters | 1 voter | 1 voter | 1 voter



Copeland

- Winner(s): candidate(s) with the largest number of outgoing edges
- That is, the ones winning in the most number of pairwise competitions



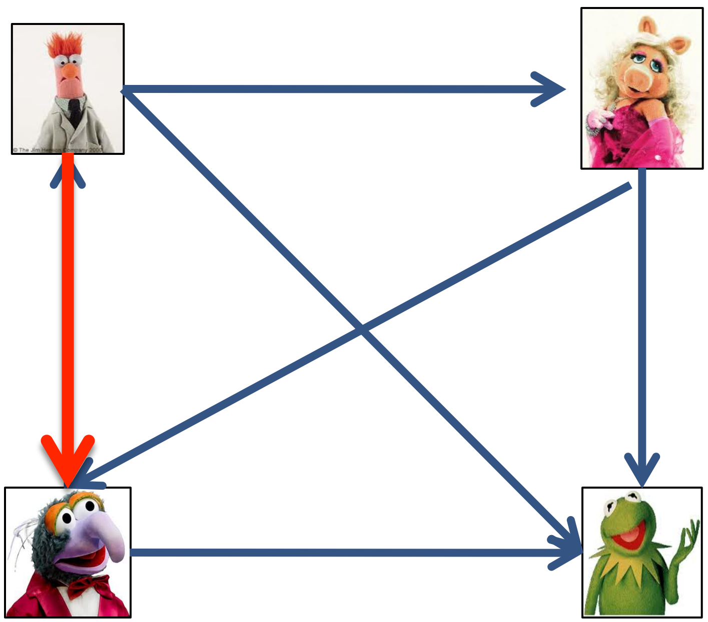
- Tie-breaking plays and important role

Monotonicity

- Intuitively, when a winner receives increased support, she should not become a loser.
- If x is a winner given a ballot b, then x wins in all other ballots obtained from b by moving x higher in the voters preferences.
- Also known as Maskin monotonicity

Copeland is monotonic

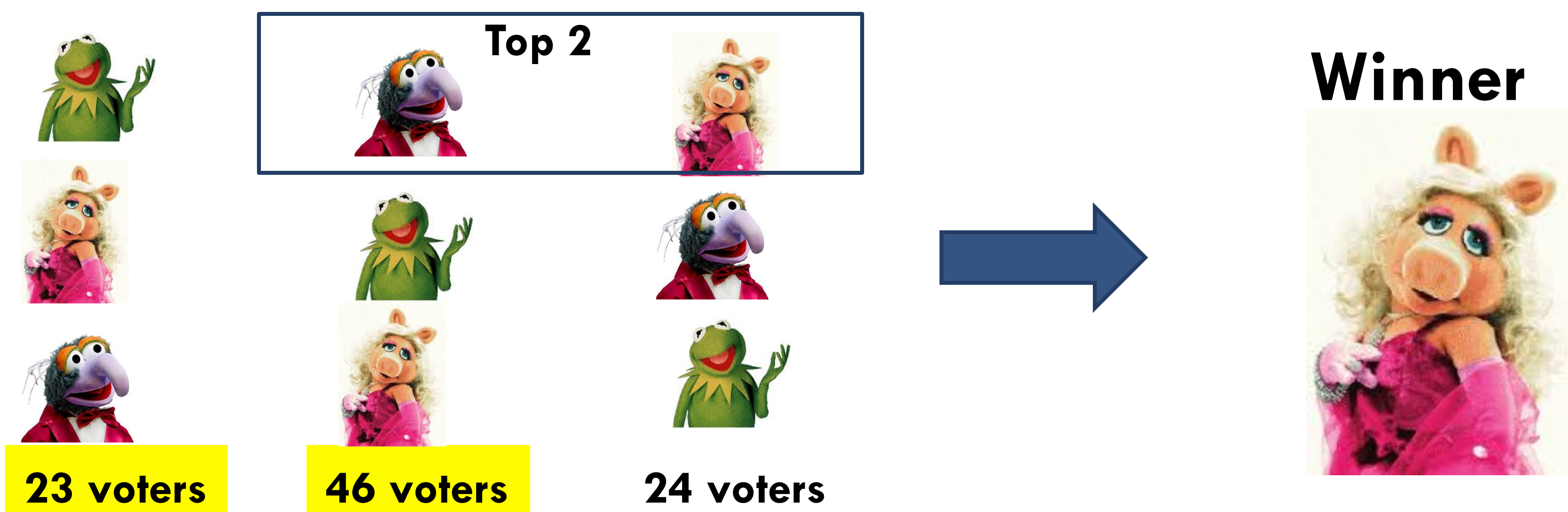
- Moving a candidate up in the rankings can only increase the number of pairwise competitions he wins



Plurality with runoff is not monotonic

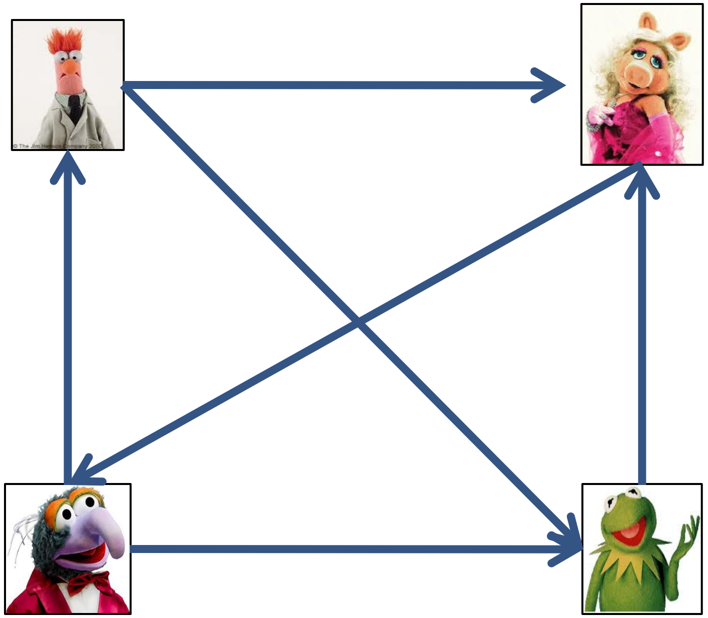
- Plurality satisfies monotonicity, but with run-off it does not

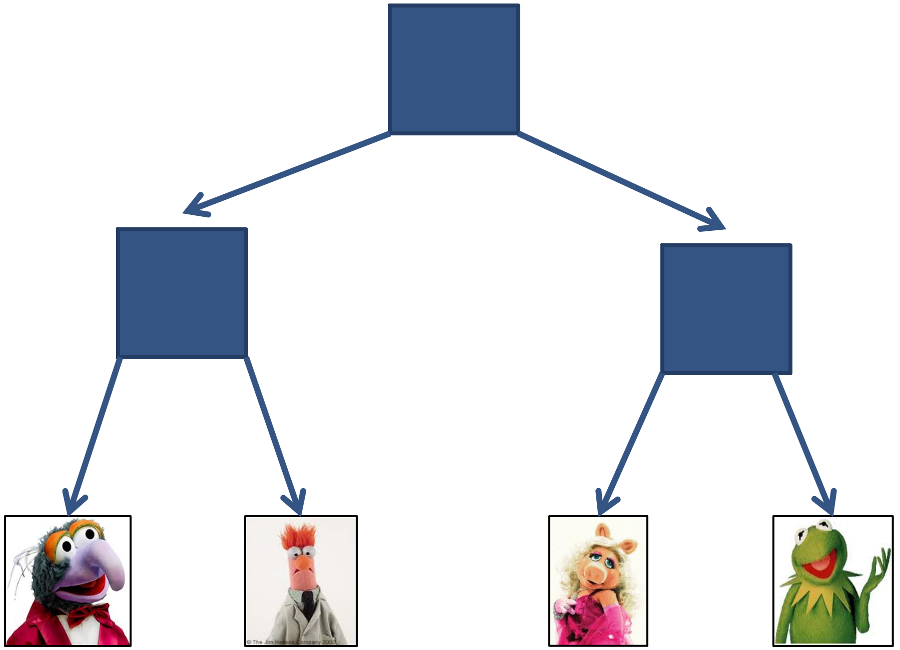
4 voters of the 1st group raise Gonzo to the top and join the 2nd group



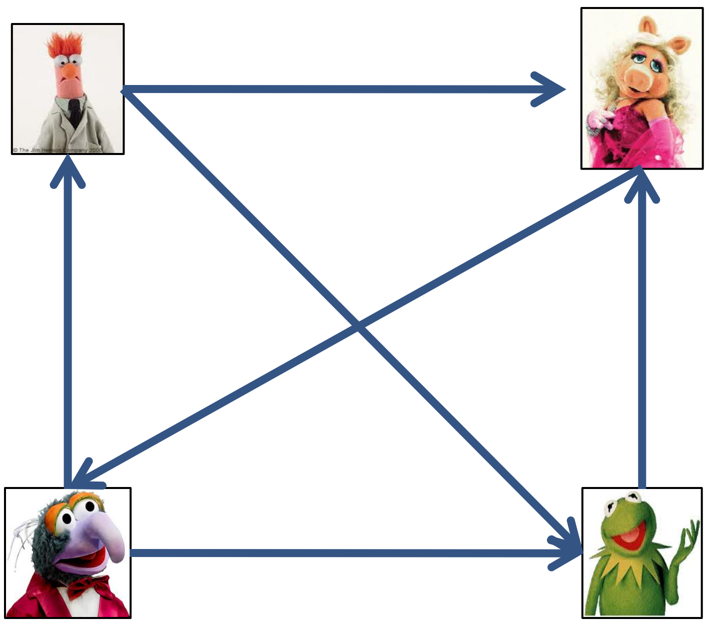
Cup rule

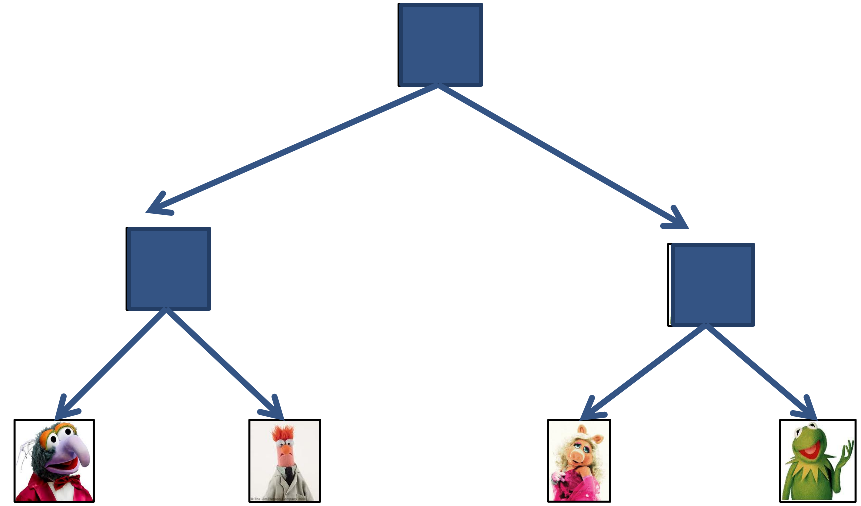
- An agenda of pairwise competitions is given via a binary tree where the leafs are candidates and each node corresponds to the winner of a pairwise competition
- The winner is the candidate associated with the root

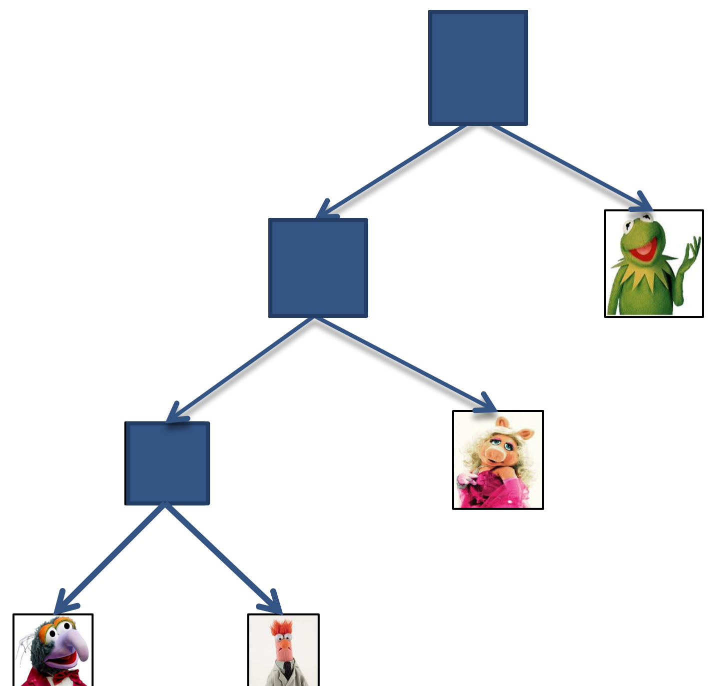




Different agenda, different winner







Complexity of computing the winner

- For the rules we have considered so far, the procedure that gives the winner is polynomial in the size of the profile O(|voters|*|candidates|).
- More formally consider the following decision problems:

R-WINNER:

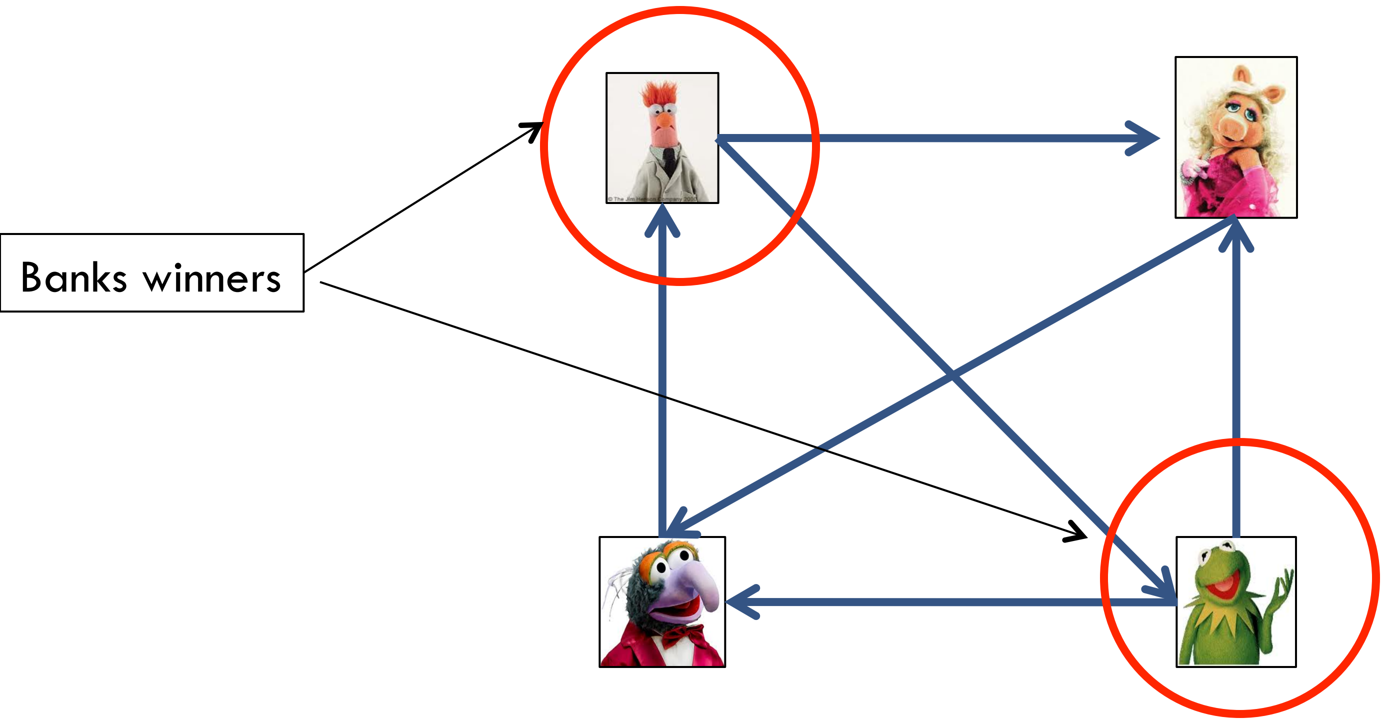
Given voting rule R, profile p of n voters on m candidates, and a candidate x, is x a winner using R?

- TH: **R-WINNER is in P when R∈{Plurality,Plur. w. run-off, STV, Borda}**
- **Proof:**

1. Compute the winner (polynomial time)
2. Check if it is x

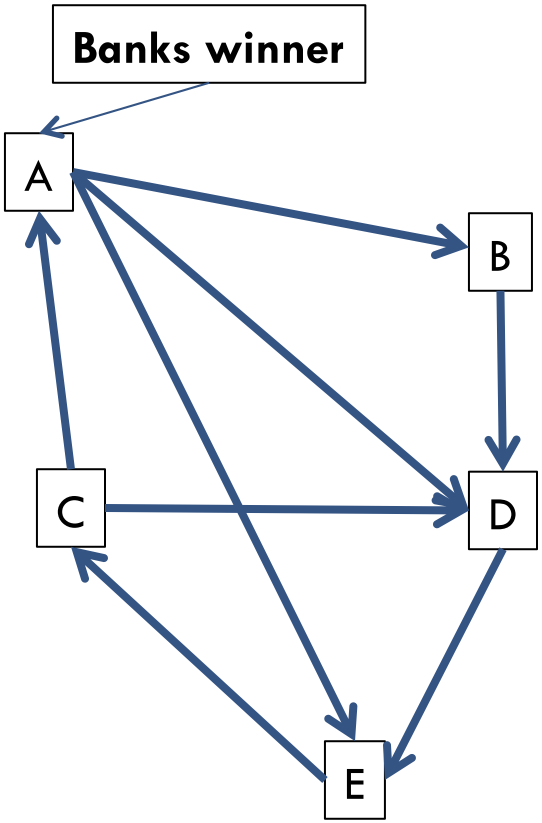
Banks

- A candidate x is a winner if it is a **top element** in a **maximal acyclic subgraph** of the majority graph.



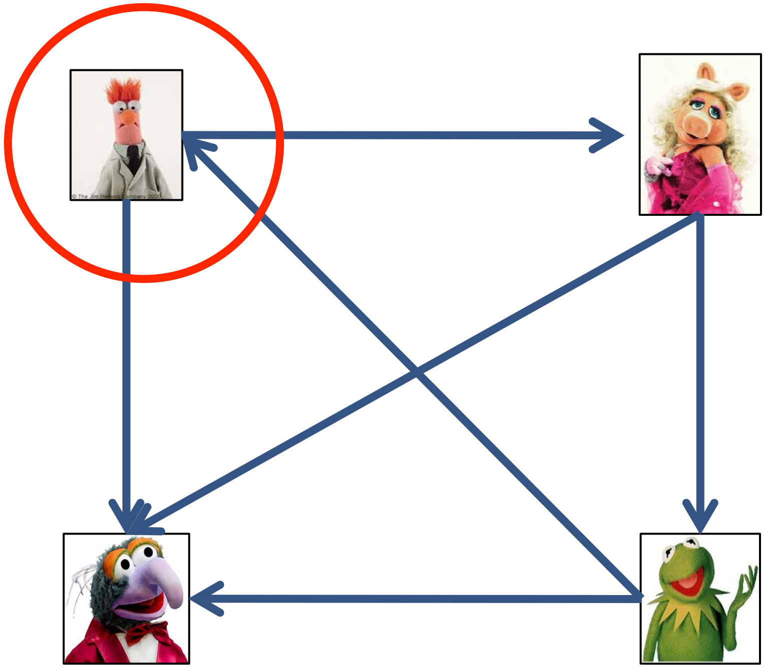
Banks rule

- TH1: **Banks-WINNER is NP-complete**
- **Proof**
  1. Compute the majority graph (polynomial time)
  2. NP: polynomial witness is a maximal acyclic subgraph
  3. NP-hardness: reduction from GRAPH 3-COLORING
- TH1 implies that computing all the Banks winners is NP-hard
- **TH2: Computing a Banks winner is easy**
- Proof:
  1. Order the candidates,
  2. start with the set with just the first candidate and then
  3. try to add 1 by 1 the others while preserving acyclicity



Slater

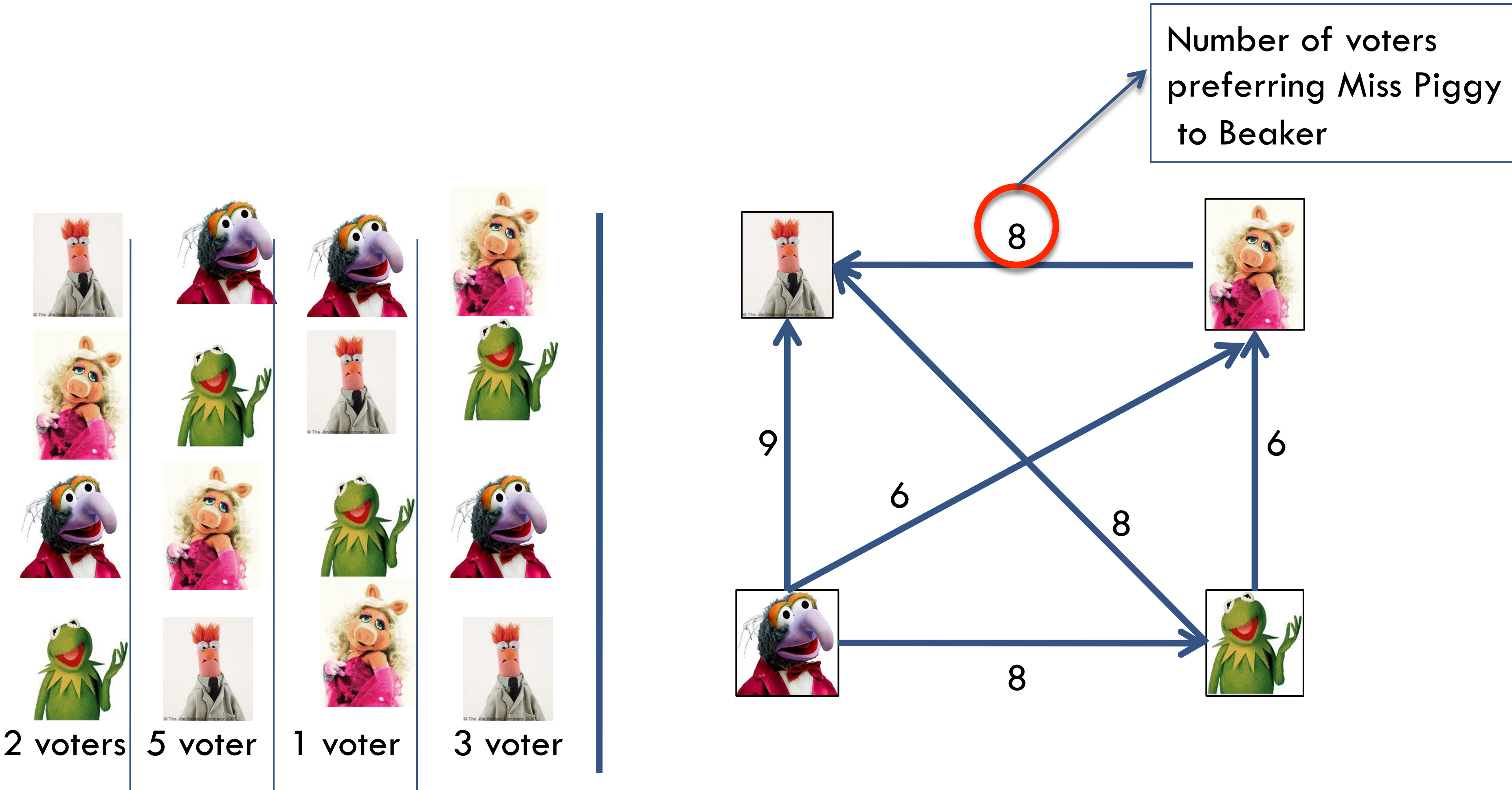
- Slater ranking: a linear order over the candidates which disagrees with the majority graph on the smallest set of pairs
  - NP-hard to compute
- Slater winner: top candidate of a Slater ranking
- NP-hard to compute



Weighted-majority-graph-base rules

Weighted majority graph

- Arcs are labeled with the entity of the majority



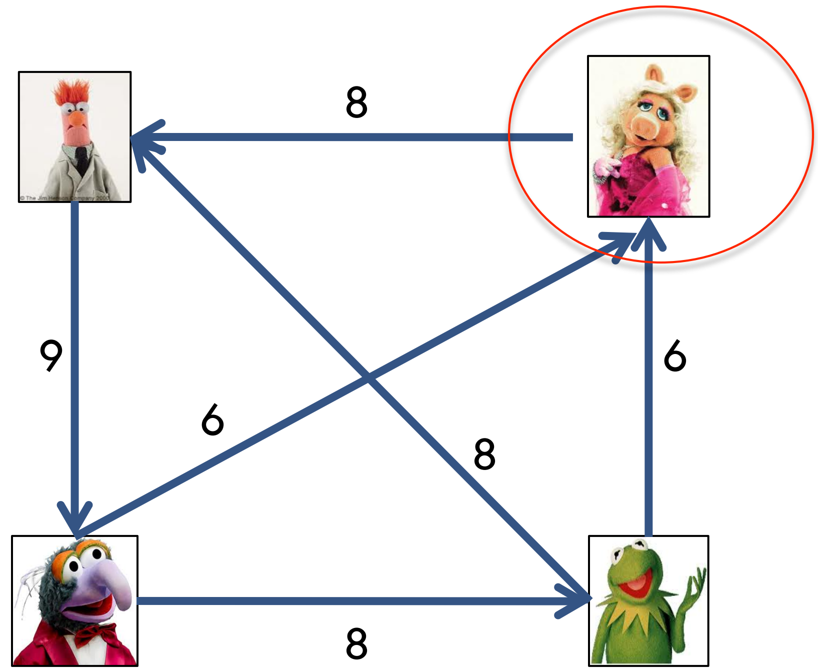
Minimax (1)

- Selects the winner with the smallest biggest pairwise defeat
- For each ordered pair of candidates (x,y), N(x,y)=number of voters that prefer x to y

- Minimax score: $S_x = \max_{y \neq x} N(y,x)$
- Minimax winner x: minimal S_x score

Minimax (2)

- In the weighted majority graph: with the smallest maximum weight on incoming arcs



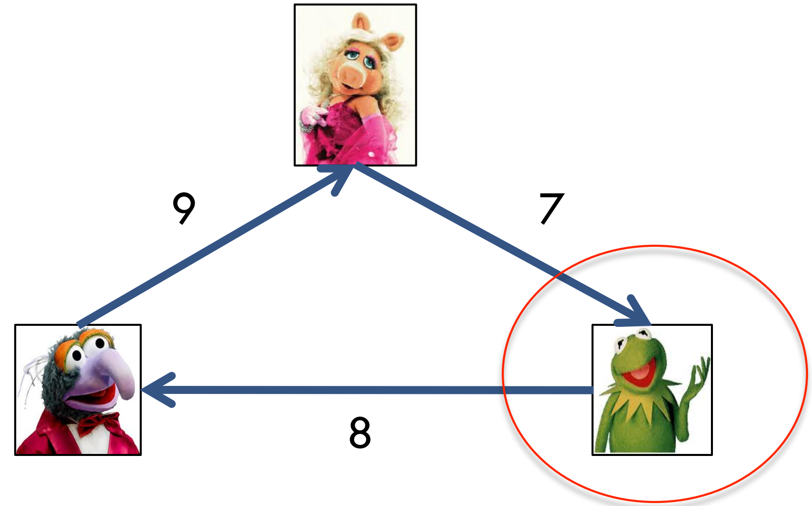
Independence of Irrelevant Alternatives

- A voting procedure is independent of irrelevant alternatives (IIA) if, whenever **x is a winner** and **y is not** and the relative **ranking of x and y does not change** in the ballots, then **y cannot win** (independently of any possible changes wrt. other, irrelevant, alternatives).

Minimax violates IIA

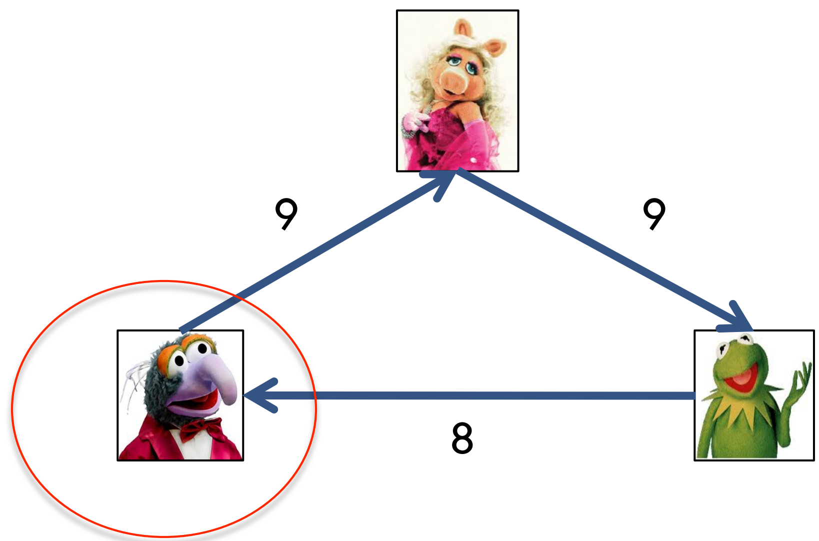
2 votes 4 votes 3 votes 4 votes

13 voters



2 votes 4 votes 3 votes 4 votes

13 voters

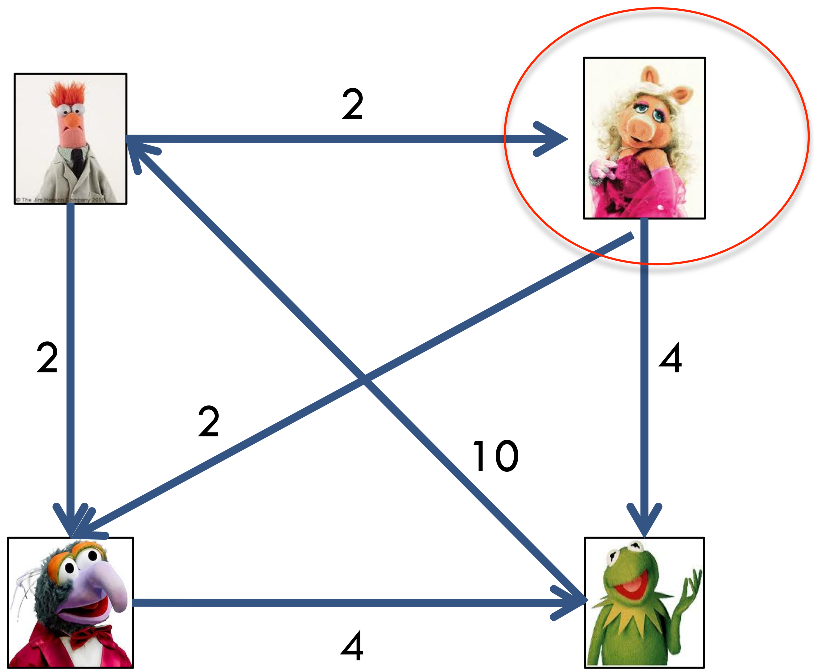


Kemeny(1)

- Closest social preference on average to the individual preferences
- Given
  - r: linear order over the candidates (aka ranking)
  - v: linear order representing the preferences of a voter
  - a,b: two candidates
- We define
  - $d_{ab}(r,v)=1$ if r and v disagree on the order of a and b
  - $d_{ab}(r,v)=0$ otherwise
- A Kemeny ranking r minimizes $\Sigma_{ab} \Sigma_{v} d_{ab}(r,v)$

Kemeny(2)

- In the weighted majority graph: minimizes the total weight of the inverted edges



Condorcet-consistent rules that use full ballot information

Bucklin

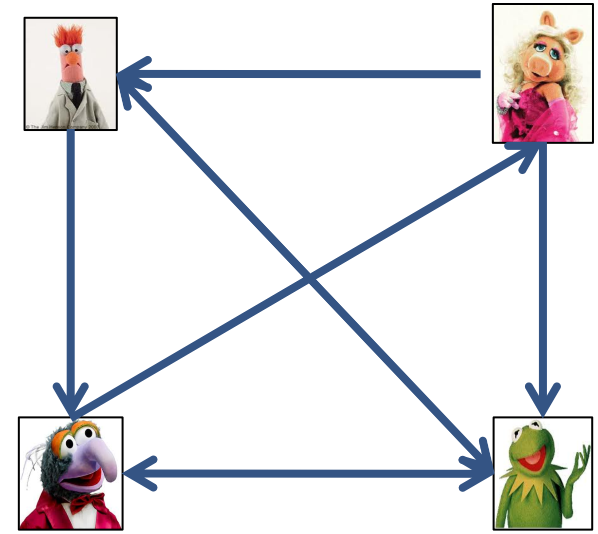
- Ballot: linear order over candidates
- Consider only first votes. If a candidate has majority→ elected
- Add second choices, and so on, until one candidate has the majority

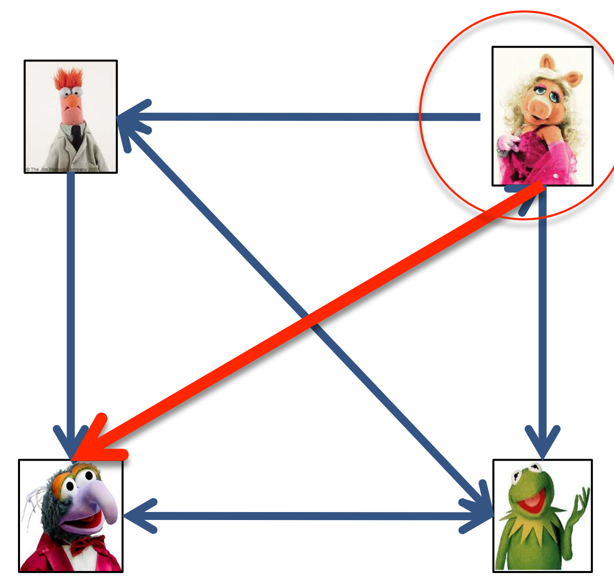
Bucklin: example

Dodgson rule

- Ballot: linear order over the candidates
- Winner: the candidate that can be made a Condorcet winner with the fewest number of inversions in the profile

Dodgson: example

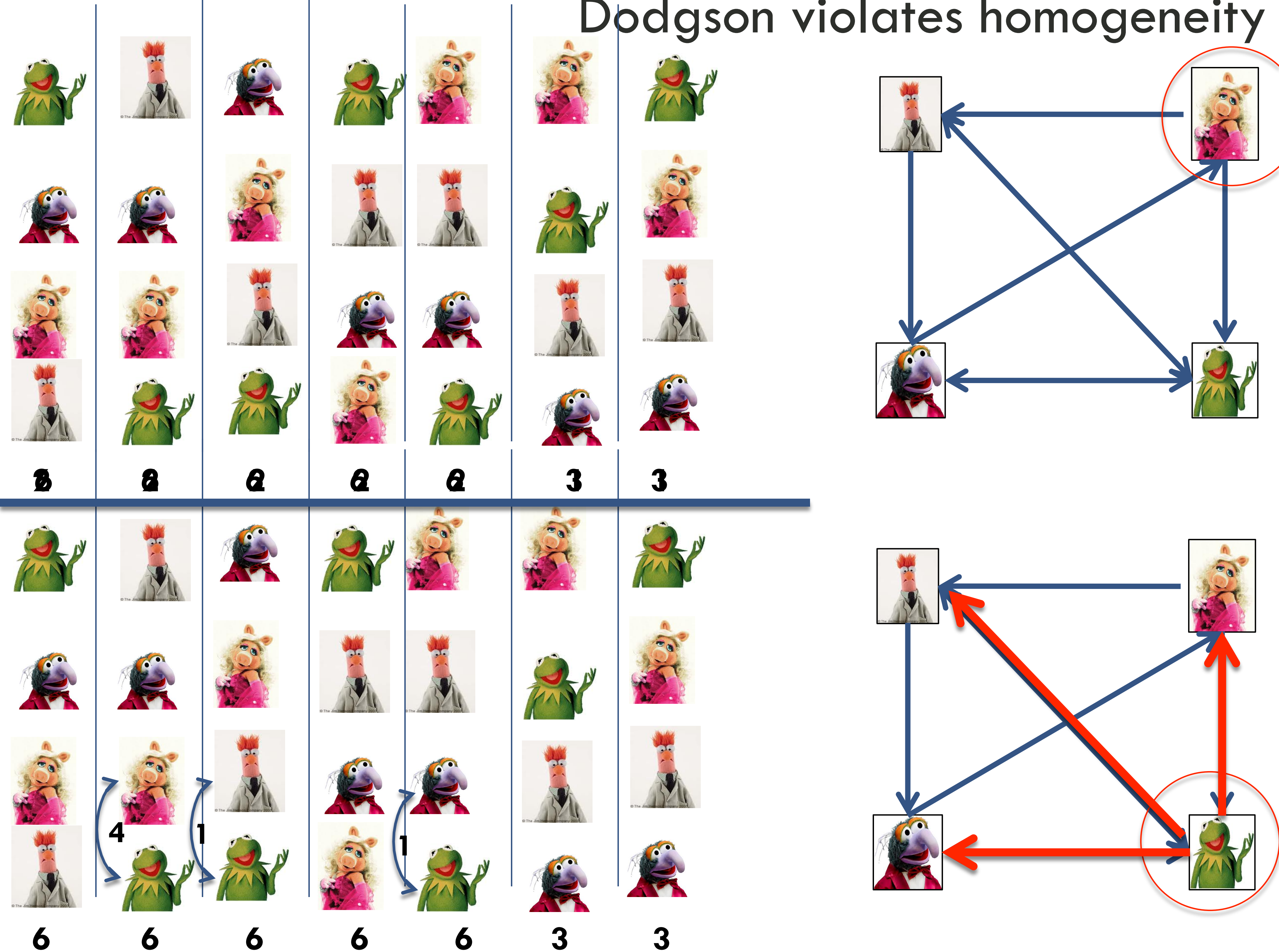




Homogeneity

- A voting rule is homogeneous if uniformly replicating voters does not affect the election outcome
- Uniformly duplicating: multiply by a constant factor greater than 0

Dodgson violates homogeneity



Range voting

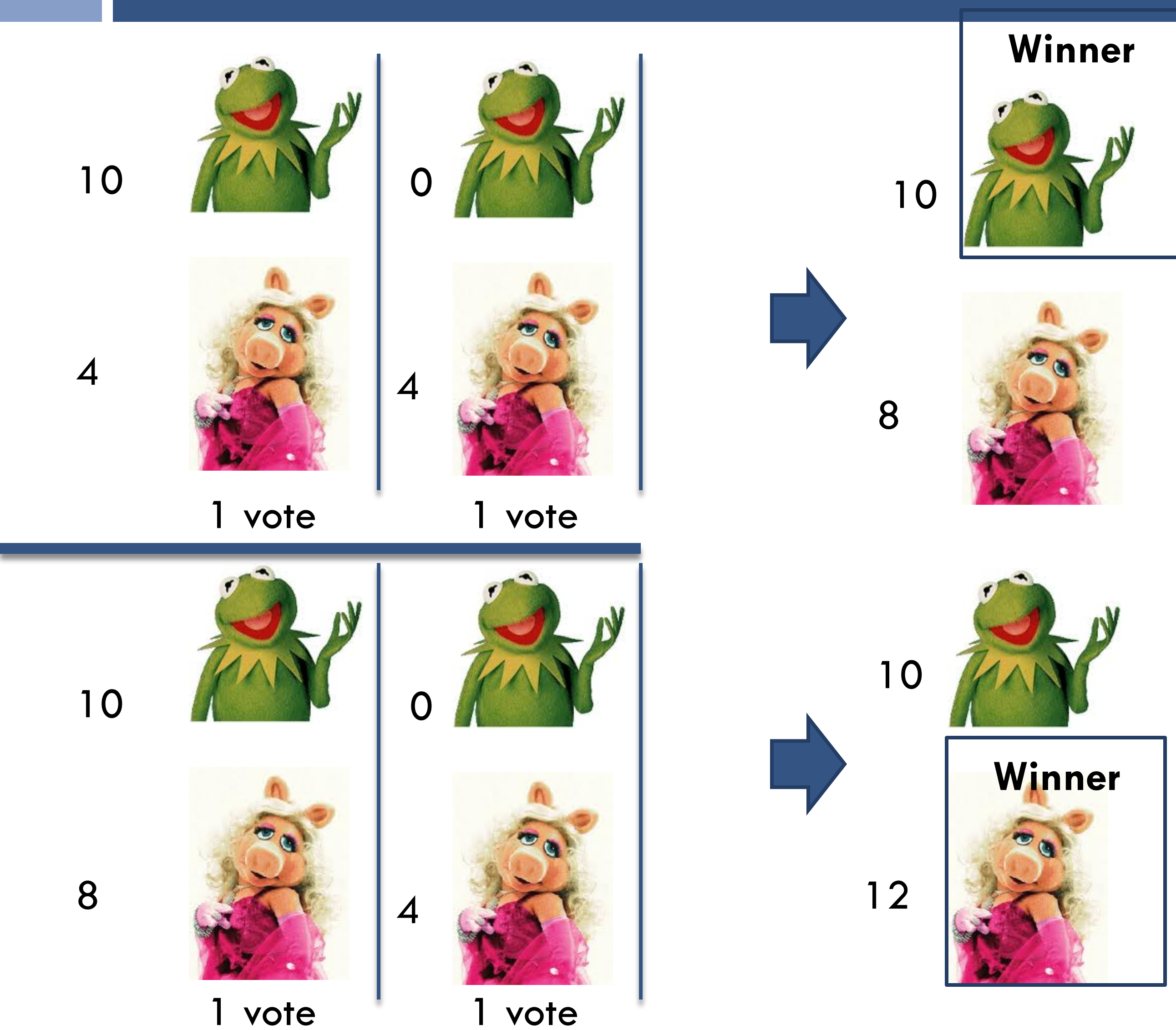
- Voters assign to each candidate a score in an interval (e.g. [0,99])
- Scores are summed
- The candidate with the highest score wins

Range voting: example

Later-no-harm

- If in any election a voter giving an additional ranking or positive rating to a less preferred candidate cannot cause a more preferred candidate to loose

Range voting violates later-no-harm



Which rule?

- Since there are so many rules, which one should we choose?

- Social Choice Theory gives an axiomatic answer
  1. Define several desirable properties (axioms)
  2. **Characterizitation Theorems**: show that a particular class of procedures is the only one satisfying a given set of axioms
  3. **Impossibility Theorems**: show that there exists no voting rule satisfying a given set of axioms

Characterization Theorems

Two candidates

- All the rules defined collapse to the same voting rule when there are only two candidates and behave as expected
- May's Theorem formalizes this idea

Positive responsiveness

- Whenever some voter raises a (possibly tied) winner in her ballot, then it becomes the unique winner of the election
- Weak monotonicity requires only for such a candidate to remain a winner
- Positive Responsiveness implies weak monotonicity (for voting correspondences)

May’s Theorem

TH: A voting procedure for two alternatives satisfies

- **Anonynimity**
- **Neutrality**
- **Positive Responsiveness**

If and only if it is the plurality rule (=majority).

Works also when ties are allowed in the ballots

Proof sketch of May's Theorem

- ← Plurality is anonymous, neutral, and positively responsive
- →
- Assume odd number of voters
- Anonymity + Neutrality + 2 candidates → only the number of votes matters
- A: set of voters voting for a
- B: set of voters voting for b
- Scenario 1: If $|A|=|B|+1$ then only a wins
  - Thus, by PR we have that a wins whenever $|A|>|B|$
  - Thus we are using plurality
- Scenario 2: there exist A and B such that $|A|=|B|+1$ but b wins
  - Let one voter in A switch to B
  - Thus, by PR, b still wins
  - This however contradicts the fact that now we have $|B'|=|A'|+1$ and the new profile can be obtained swapping a snd b in the previous profile
  - Thus by neutrality a should win

Reinforcement (aka Consistency)

- Split the voters into two sets
- A candidate that wins the election with both sets wins also the full election

Continuity

- Whenever a set of voters N elects a unique winner x, then for any other set of voters N' there exist a number k such that N' together with k copies of N will elect only x

- Weak requirement

Young's Theorem

TH: A voting procedure satisfies

- **Anonynimity**
- **Neutrality**
- **Reinforcement**
- **Continuity**

If and only if it is a positional scoring rule.

Characterization via consensus and distance

- Rationalization of voting procedures
- **Consensus class:** subset of profiles with a clear set of winners
- **Distance:** measures how different are two profiles
- **Induced rule:**
  1. Fix a consensus class
  2. Fix a distance measure
  3. for each profile, compute the closest profile in the consensus class according to the distance measure and elect the corresponding winner(s)

Consensus classes

- **Condorcet winner:** beats all other candidates in pairwise competitions
- **Majority winner:** there is a candidate which is ranked first by an absolute majority
- **Unanimous winner:** there is a candidate which is ranked first by all voters
- **Unanimous ranking:** all the voters have the exact same ranking (and the top wins)

Distance metrics

- **Swap distance** of two profiles b and b’: number of adjacent pairs of candidates that need to be swapped to get from b to b’
- **Discrete distance between two ballots, for example:**
  - 0 if the they are the same
  - 1 otherwise
- **Discrete distance of profile:** sum of ballots distances

Characterization results

- Dodgson rule: Condorcet winner + swap distance
- Kemeny rule: Unanimous ranking + swap distance
- Borda: Unanimous winner + swap distance
- Plurality: Unanimous winner + discrete distance

Impossibility Theorems

Non-imposition

- A voting procedure satisfies non-imposition if **each alternative** is the **unique winner under at least one ballot profile.**
- Any surjective (onto) voting procedure satisfies non-imposition. For resolute procedures, the two properties coincide.
- Any neutral resolute voting procedure satisfies non-imposition

Dictatorship

- A voting procedure is dictatorial if there exists a voter (the dictator) such that the unique winner will always be his top-ranked alternative.
- A voting procedure is **non-dictatorial** if it is not dictatorial.
- Any anonymous voting procedure is non-dictatorial

Unanimity and Pareto Condition

- A voting procedure is **unanimous** if it elects (only) x whenever all voters say that x is the best alternative.

- The **weak Pareto condition** holds if an alternative y that is dominated by some other alternative x in all ballots cannot win.
- Pareto condition entails unanimity, but the converse is not true.

Independence of Irrelevant Alternatives

- A voting procedure is independent of irrelevant alternatives (IIA) if, whenever **x is a winner** and **y is not** and the relative **ranking of x and y does not change** in the ballots, then **y cannot win** (independently of any possible changes wrt. other, irrelevant, alternatives).

Arrow's Theorem

- **TH: No voting procedure for more than 3 candidates can be at the same time**

1. **weakly Pareto**
2. **IIA**
3. **non dictatorial**

- Wow!

- Does not hold for two alternatives (majority)
- IIA is debatable (hard to satisfy)

Nobel prize in Economics 1972

Proof of Arrow's Theorem (1)

- Many versions of Arrow's Theorem
- We use Sen 1986, "decisive coalition technique"
- X set of candidates
- N set of voters
- **Decisive subset** of voters G for pair of candidates (x,y), if when voters in G prefer x to y, then y is not a winner
- **Almost decisive subset** of voters G for pair of candidates (x,y), if when only the voters in G prefer x to y, then y is not a winner

Proof of Arrow's Theorem (2)

- Proof steps

1. Weak Pareto condition = N is decisive for all pairs
2. Lemma1: G almost decisive for some (x,y) → G decisive for all (x,y)
3. Lemma2: given subset of voters G, with |G|>1, decisive for all pairs → there exists G' subset of G which is decisive for all pairs
4. Thus, by induction, there is a decisive subset of size 1(= a dictator)

Proof of Arrow's Theorem (3)

- **Pareto condition = N is decisive for all pairs**

- The *weak Pareto condition* holds if an alternative y that is dominated by some other alternative x in all ballots cannot win.

- *Decisive subset* of voters G for pair of candidates (x,y), if when voters in G prefer x to y, then y is not a winner

Proof of Arrow's Theorem's (4)

- **Lemma1: G almost decisive for some (x,y) → G decisive for all (x,y)**
- **Proof**
- Let x,y,a,b be distinct candidates
- Consider the profiles where:
  - Voters in G have : a>x>y>b
  - All others: a>x, y>b, y>x (rest unspecified)
- G almost decisive for (x,y) → y cannot win
- Weak Pareto → x cannot win and b cannot win
- Thus b loses and a wins in a situation where a>b in G independently of how a and b are raked by all others
- IIA → be will not win in any profile where a>b in G
- Thus G is decisive for (a,b)

Proof of Arrow's Theorem's (4)

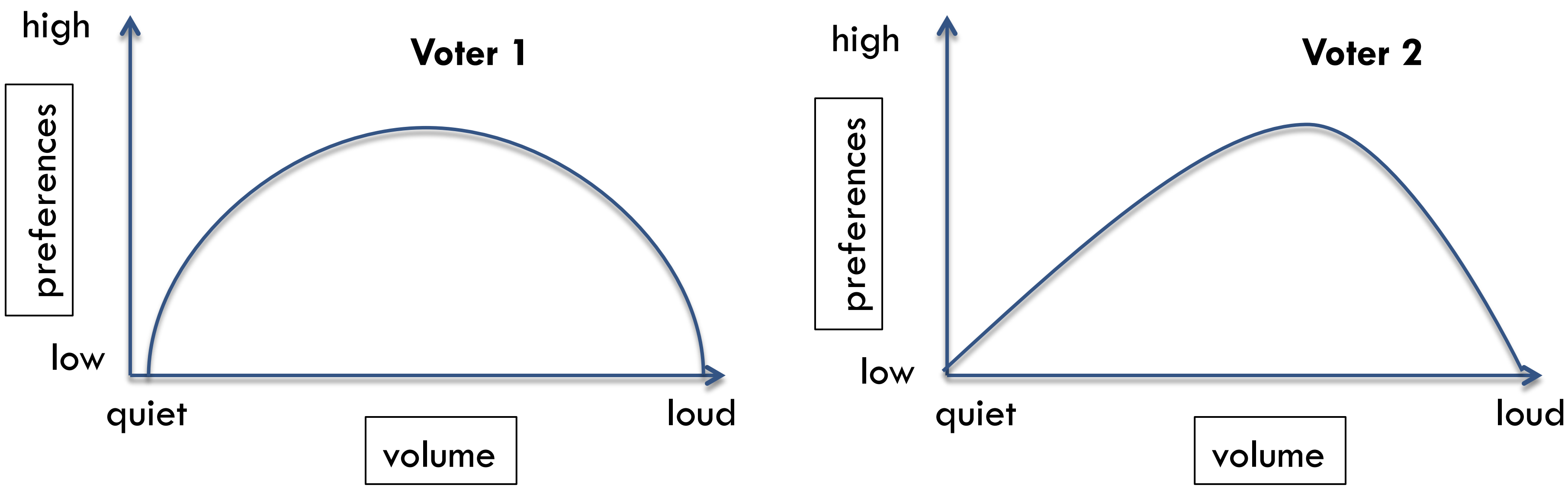
- **Lemma2 (Contraction): given subset of voters G, with |G| >1, decisive for all pairs → there exists G' subset of G which is decisive for all pairs**
- **Proof**
- Divide G into two non empty subsets: G1 and G2
- Consider the following profile:
  - Voters in G1: x>y>z
  - Voters in G2: y>z>x
  - All others: z>x>y
- G decisive → z cannot win → either x wins or y wins
- Case 1: x wins
- Note that only G1 has x>z
- IIA → z will not win in any profile where G1 has x>z
- Thus, G1 is almost decisive for (x,z)
- From lemma 1 G1 is decisive for all pairs, and its cardinality is smaller than the cardinality of G.
- Case 2: y wins
- Note that only G2 has y>x
- Same as above G2 is decisive for all pairs and its cardinality is smaller than the cardinality of G.

Escaping Arrow's Theorem

- There are cases that allow to escape the reach of Arrows theorem
- For example, range voting satisfies all three axioms
- Arrow's theorem does not apply to range voting since the input is a not a profile composed of linear orders
- Another possibility is to put restrictions on the ballots

Single Peaked Preferences

- There exist a fixed linear ordering of the candidates such that the preferences of all individuals are single-peaked w.r.t. this ordering



Two voters deciding at which volume to listen to the radio

Black's Possibility Theorem

- **TH: If a profile of ballots from an odd number of voters dealing with more than two alternatives has single-peaked preferences in some ordering of the alternatives, then the social preference relation *P* is transitive (the majority graph is acyclic).**
- Thus, the majority rule is weakly Pareto, IIA and non dictatorial

Sen's Theorem generalizes Black's Theorem

- A profile of ballots is **coherent** if for any three alternatives, at least one of the three, which we call *x*, satisfies at least one of these conditions:
  - No voter ranks *x* above both of the other two alternatives.
  - No voter ranks *x* between the other two alternatives.
  - No voter ranks *x* below both of the other two alternatives.

- **TH If a profile of ballots from an odd number of voters dealing with more than two alternatives is coherent, then the social preference relation is transitive (=no cycles in the majority graph).**

Monotonicity

- Intuitively, when a winner receives increased support, she should not become a loser.
- If x is a winner given a ballot b, then x wins in all other ballots obtained from b by moving x higher in the voters preferences.
- Also known as Maskin monotonicity

The Muller Satterthwaite theorem

- Monotonicity turns out to be (desirable but) too demanding:
- **TH: No resolute voting procedure for at least 3 alternatives can be**

1. **non-imposing (surjective),**
2. **monotonic,**
3. **and non-dictatorial**

What happens if we have partial orders

- In many AI frameworks alternatives are partially ordered rather than totally ordered
  - Candidate domain of large size
  - Uncertainty
  - Combinatorial structure
- Do we escape impossibility results if we allow voters to relax their ordering from total to partial orders (thus allowing incomparability)?
- Unfortunately not. Arrow's and Muller-Satterthwaite theorem can be extended to partial orders

[Pini, Rossi, Venable,Walsh, 2009]