Interpretable Preference Learning: A Game Theoretic Framework for Large Margin On-line Feature and Rule Learning

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Abstract

In this work, a preference learning problem is cast into a two-players zero-sum game. During the learning process, the maximum margin hypothesis is incrementally obtained with the inclusion of new useful features. A game theoretical analysis is used to demonstrate the convergence of the algorithm. Leveraging on the natural analogy between features and rules, the resulting models can be easily interpreted by humans.

Preference Learning as a two-players zero-sum game

Label ranking tasks consider a set of pairwise preferences $y_i \succ_{\mathbf{x}} y_j$ (label y_i preferred to label y_j, for a pattern $\mathbf{x}$). The margin of a hypothesis $\mathbf{w}$ on a given preference is computed by $\rho(\mathbf{z}) = \mathbf{w}^\intercal\mathbf{z}$, where $\mathbf{z}$ is a convenient representation of the preference.

PRL learns the maximal margin hypothesis. The maximization of the minimum margin on the training preferences is formulated as a two-players zero-sum game where:

- rows of the game matrix $\mathbf{M}$ correspond to training preferences;
- columns of the game matrix $\mathbf{M}$ correspond to preference-feature pairs;
- entries $\mathbf{M}_{i,(j,f)} = \mathbf{z}_i[f]^\intercal\mathbf{z}_j[f]$, where $\mathbf{z}[f]$ indicates the feature f of the preference $\mathbf{z}$;
- the value of the game V^* is the optimal margin, computed by

$$V^* = \min_{\mathbf{p}} \max_{\mathbf{q}} \mathbf{p}^\intercal\mathbf{M}\mathbf{q},$$

where $\mathbf{p}$ and $\mathbf{q}$ are mixed strategies for the row and column players.

Finding the saddle-point of such huge game matrices using off-the-shelf game theoretical methods is computationally expensive. PRL iteratively considers small subsets of columns, in such a way that, at each iteration, the sub-optimal computed solution becomes closer and closer to the optimal one (Figure 2).

Online feature/rule generation

One of the most important steps of PRL is column generation. In our experiments we employed two feature generation schemes: polynomial feature generation, and rule generation. In particular, rules are very useful when interpretability is desired.

PRL: Preference and Rule Learning algorithm

Theoretical analysis demonstrates that, at each PRL iteration, the value of the game increases and it is upper bounded by the optimal margin:

$$V_t \leq V_{t+1} \leq \cdots \leq V^*$$

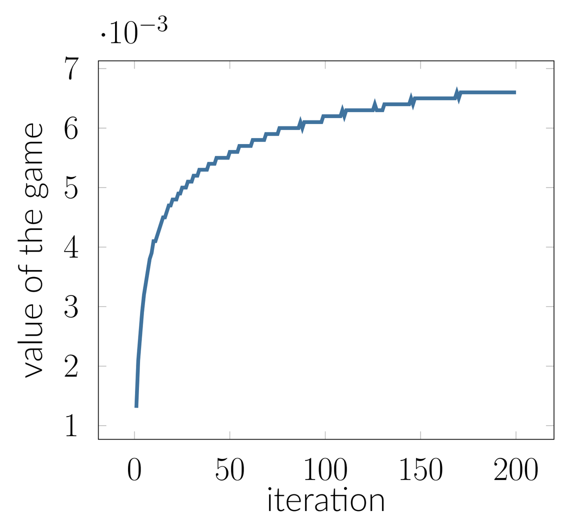

Figure 1. Empirical assessment of the increase of the value of the game w.r.t. the iteration of PRL on the `mnist` dataset.

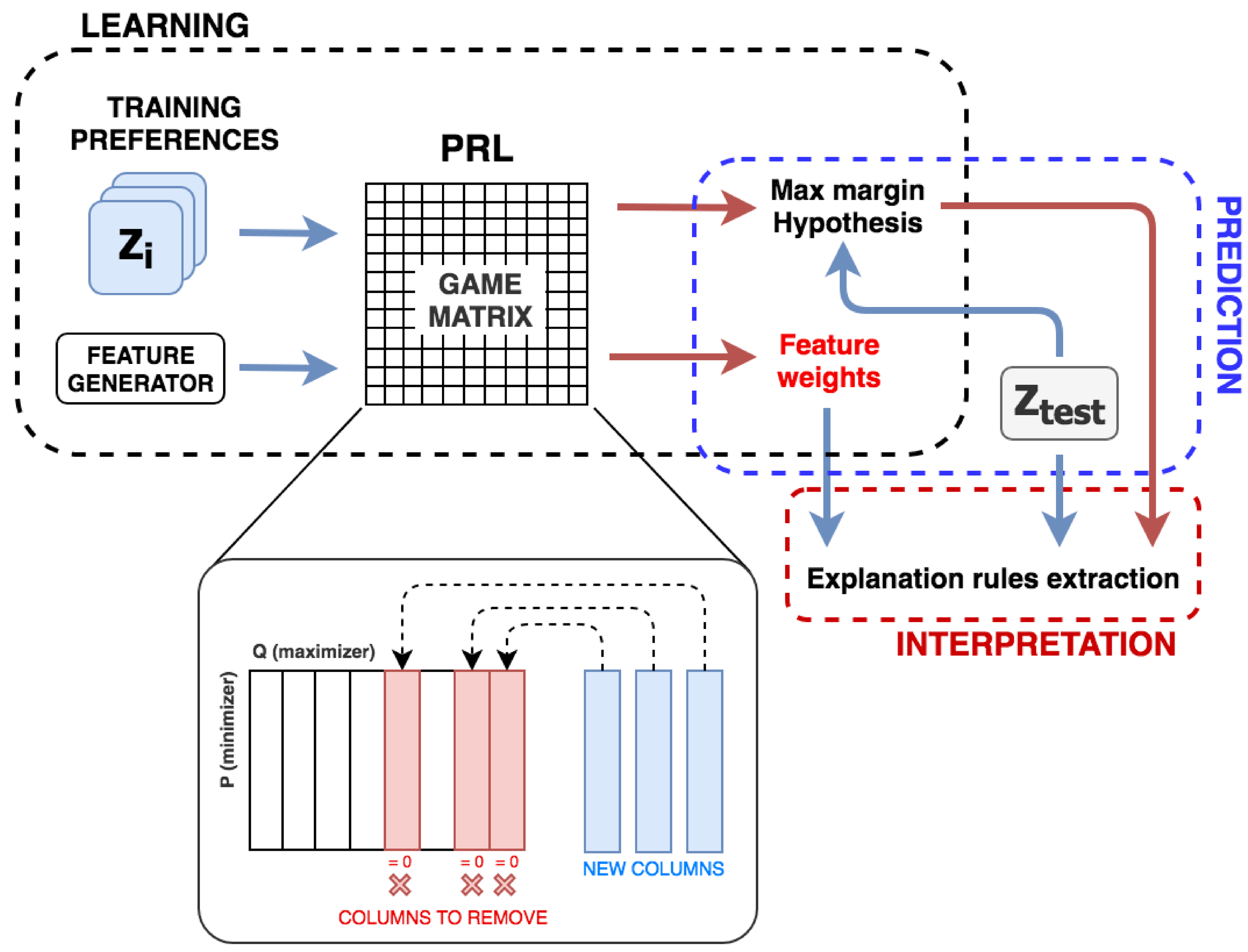

Figure 2. Schema of the PRL algorithm. The zoomed part emphazises columns which are not part of the current strategy are substituted with newly generated columns (preference-feature pairs).

Visual interpretation

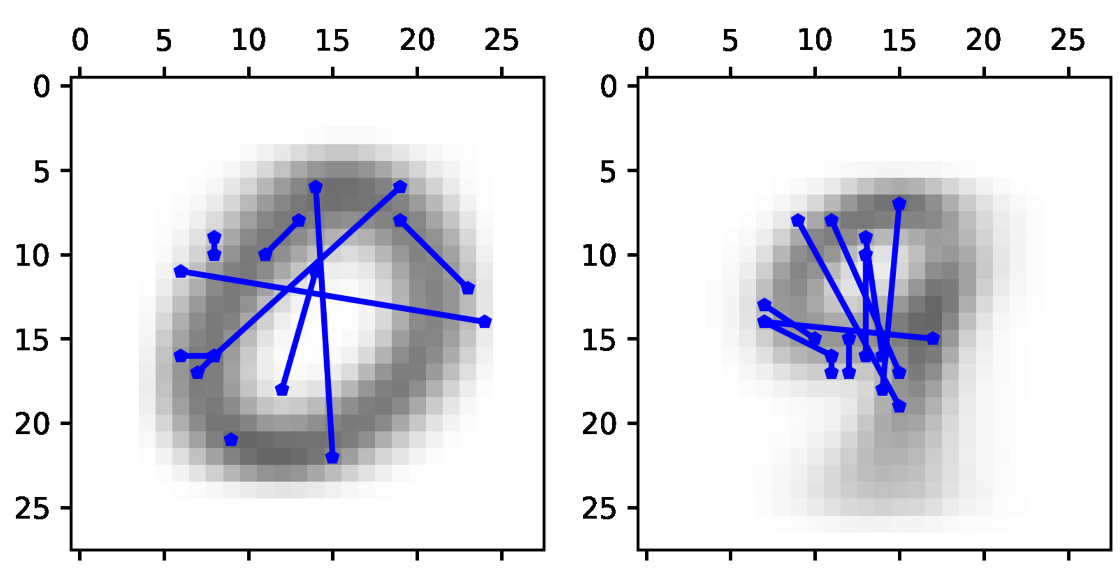

Figure 3. Visualization of the most relevant polynomial features of degree 2 in classifying a 9 w.r.t. a 0, and viceversa. PRL discriminates a 0 from a 9 by looking at the "big" curvature for 0, and the smaller one for 9.

Classification

PRL successfully identifies the explanation rules in the `poker` dataset.

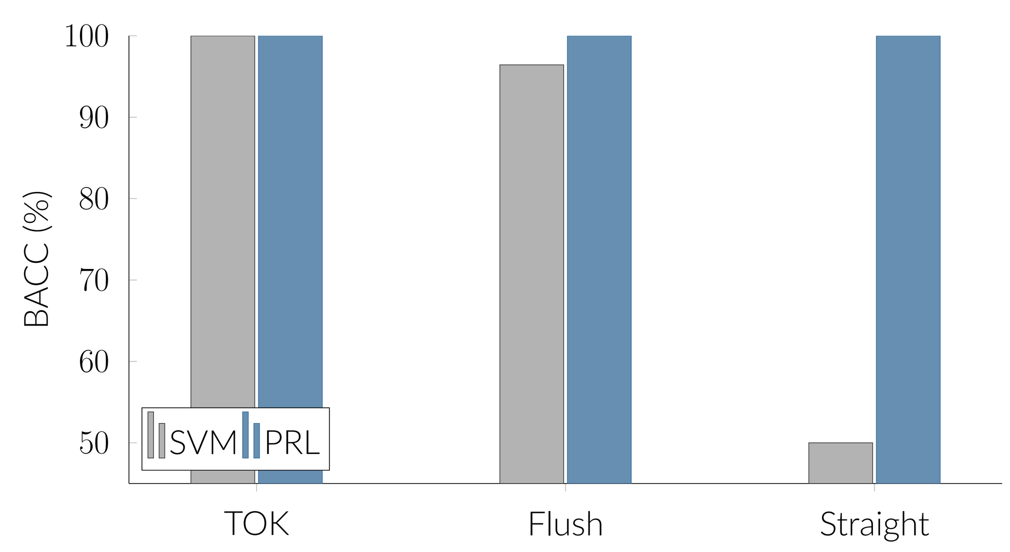

Figure 4. Balanced accuracy of PRL against SVM on the `poker` dataset on three different classification tasks: Three Of a Kind (TOK), Flush, and Straight.

Rule extraction

The most relevant features/rules can be used to explain the decision. In `breast-cancer` a rule can be: if the *clump thickness* ≤ 6 and the *Normal Neucloli* ≤ 8, then the tumor is *benign*.

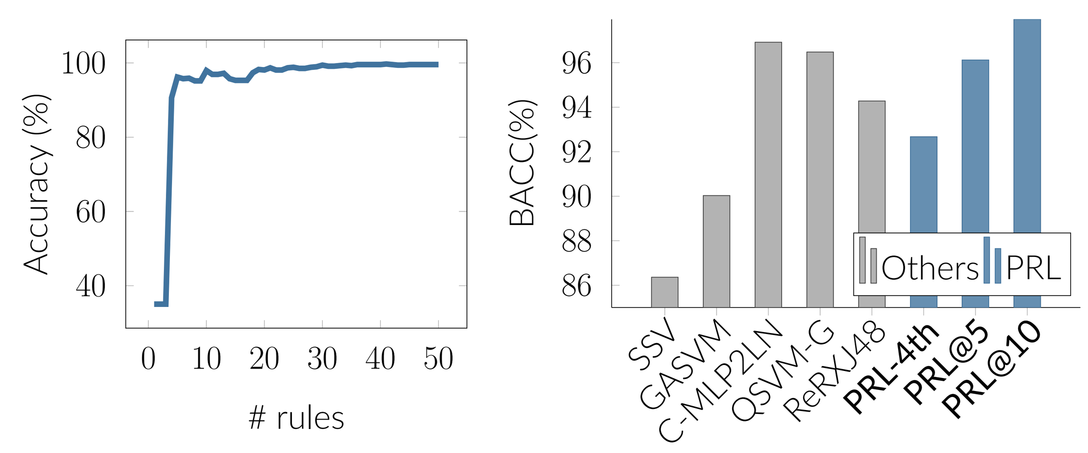

Figure 5. (left) Plot of the accuracy w.r.t. the number of considered rules during classification. (right) Balanced accuracy of the extracted rules of PRL against other rule extraction algorithms.

Feature selection

The feature selection capability of PRL makes it suitable for dealing with datasets with many features.

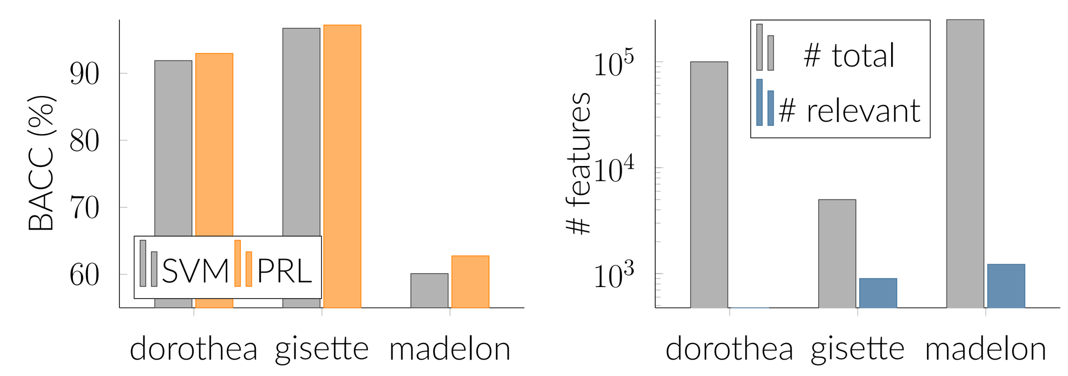

Figure 6. (left) Classification performance of PRL and SVM on three datasets with thousands of features. (right) Number of relevant features extracted by PRL w.r.t. the total number of features.