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An Accelerated Algorithm for Navier-Stokes Equations

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INTRODUCTION

The numerical approximation of the steady-state Navier-Stokes equations requires a large amount of computational time which is related to the total number of iterations, the size of the problem and the used algorithm.

So, the convergence rate may become slow and it is interesting to apply acceleration techniques to improve the overall iterative scheme.

In this work, the **Characteristic-Based-Split** (CBS) scheme is combined with the **Minimum Polynomial Extrapolation** (MPE) method to solve **Navier-Stokes problems**.

THE EXTRAPOLATION TECHNIQUE

There are no general guidelines to accelerate the convergence of evolutive problems that converge to the stationary solution. So, we adopt the following strategies:

- We **select the variables** to check where the extrapolation procedure should be done. Regardless this choice, all the solution variables are extrapolated.

- We **skip the first iterations** of the iterative process since they are, reasonable, the most far away from the stationary solution and, then, not so good to perform some extrapolation step.

- We **collect consecutive iterations** characterized by the fact that the maximum error for each variable between two consecutive iterations occurs at the same mesh nodes. This **heuristic** like to mimic the behavior of iterative solutions near stationary points where the shape of the solutions change small between two consecutive iterations. As a consequence the error between consecutive iterations are proportional and hence it is reasonably that it appears at the same nodes of the mesh. This way to check the extrapolation point is very less, memory and time, consuming.

- We **perform extrapolation** only on the **last part of the collected iterations**. This has two reasons:
  1) maintaining the number of extrapolation vectors and thus the CPU time low in solving of the MPE algorithm.
  2) checking assessment of the extrapolation criteria inside a more long sequence in order to avoid to collect less significative vectors for the extrapolation.

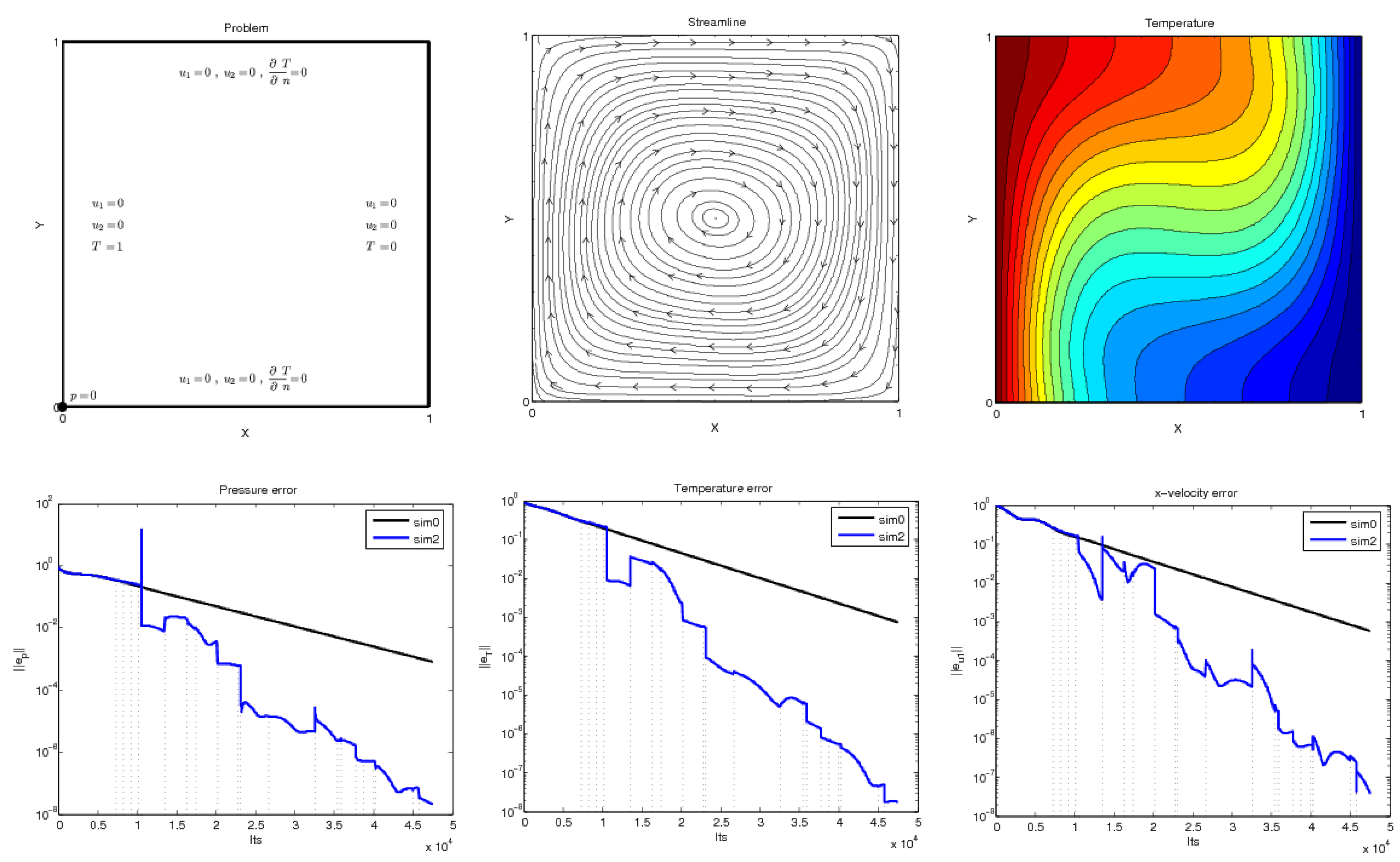
- Finally, since the extrapolated solution, typically, is not a feasible solution for the evolution problem (i.e., it does not satisfy the PDE equations), we give some iterations to **relax towards a feasible solution** before looking back again to original sequence for finding another starting point before collecting further iterations.

REFERENCES

- Venturin M., Russo M.R., Bertelle R.: **An accelerated algorithm for Navier-Stokes equations** (submitted)
- Morandi-Cecchi, M., Venturin, M.: **Characteristic-based split (CBS) algorithm finite element modelling for shallow waters in the Venice lagoon.** Int. J. Numer. Meth. Engrg. 66 (10), 1641-1657 (2006)
- Bertelle R., Russo, M.R.: **An approach to the Gummel map by vector extrapolation methods.** Numer. Algor. 45, 331-343 (2007)

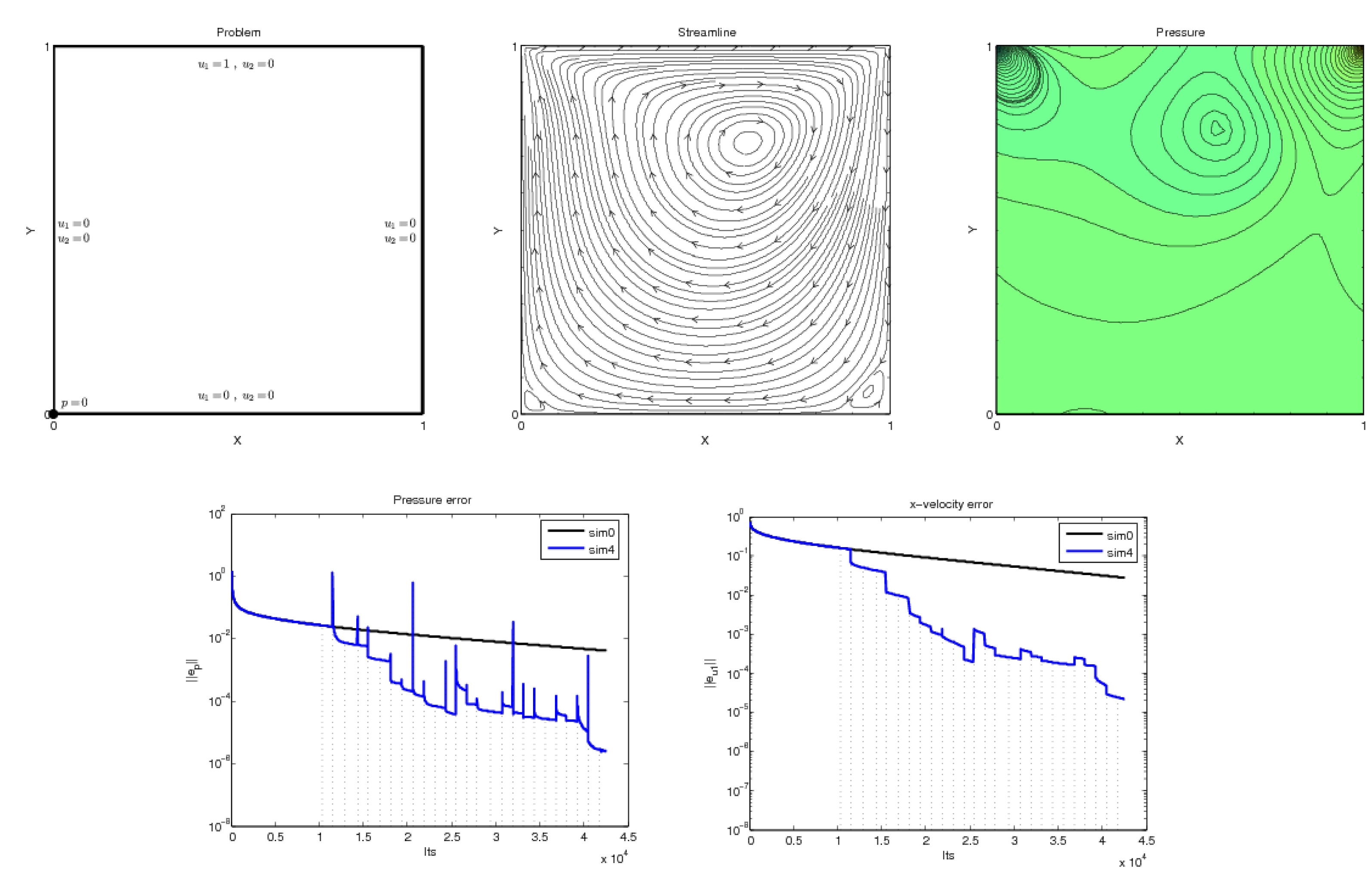
BUOYANCY-DRIVEN CONVECTION PROBLEM

In this problem the Navier-Stokes equations are coupled with the temperature equation. The local temperature difference induces a local density difference within the fluid and this produce the fluid motion.



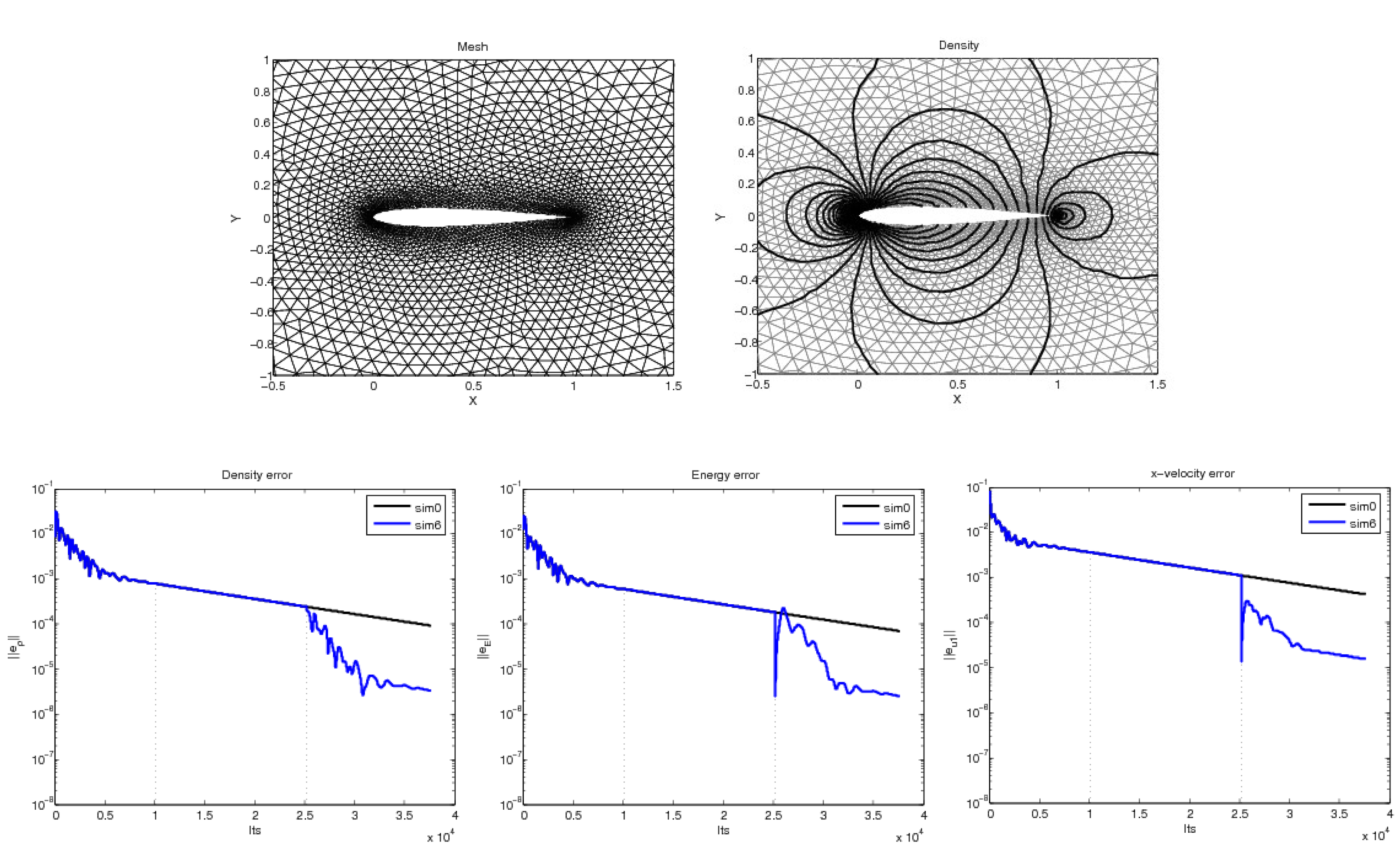
LID-DRIVEN CAVITY PROBLEM

This problem considers a square cavity domain with no-slip velocity conditions on the bottom and on the side walls, and with the top-lid moving at a given horizontal velocity.



NACA0012 AIRFOIL PROBLEM

This problem refers to an inviscid compressible flow over a NACA0012 airfoil test problem at Mach number 0.25.



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