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An acceleration method for the Gummel map

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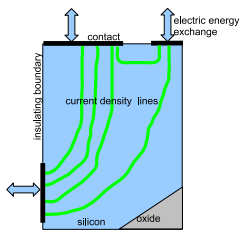
INTRODUCTION

Electronic devices are micrometer scale length regions filled with different materials where charged particles interact accordingly to the Van Roosbroeck model.



Van Roosbroeck model

$$\begin{cases} \lambda^2\Delta\psi &= n-p+C \quad (\mathrm{PS}) \\ \nabla\cdot\vec{j}_n &= R \quad (\mathrm{EC}) \\ \nabla\cdot\vec{j}_p &= -R \quad (\mathrm{HC}) \end{cases}$$

$$\begin{cases} \begin{cases} \vec{j}_n &= \nabla n - n\nabla\psi \\ \vec{j}_p &= \nabla p + p\nabla\psi \end{cases} \\ R = \frac{np-n_i^2}{\tau_n(p+n_i)+\tau_p(n+n_i)} \end{cases}$$

In the Van Roosbroeck model λ is a given scaling constant and

- n, p are the free (to move inside the semiconductor material) negative and positive carrier concentrations (electron and hole, respectively) and $\vec{j}_n$, $\vec{j}_p$ are the corresponding current densities;
- ψ is the electrostatic potential;
- C and R are given functions, the doping and the generation-recombination rate.

The doping function describes the distribution of the free charges inside the semiconductor. Positive value of C corresponds to n-silicon, rich of electrons, and negative one to p-silicon, rich of holes.
Given C and boundary conditions, the solution of the Van Roosbroeck model with respect to the unknowns n, p and ψ cannot be done, in general, in an analytic manner but needs the use of computer.
The Gummel map is a common way to solve the Van Roosbroeck model. Basically, at any step, this iterative algorithm decouples each equation from the remaining two and solves it alone with respect of one of the unknowns. It may require a large number of iterations to obtain the solution. Thus, it is interesting to apply some acceleration technique to improve the rate of convergence of the overall iterative scheme which, looks as

ACCELERATED GUMMEL MAP

guess $\psi^{(0)}, \phi_n^{(0)}, \phi_p^{(0)}$
$k = 0$
while (some stop criterion is not matched)

1. $-\nabla\cdot(\epsilon\nabla\psi^{(k+1)}) = q[p^{(k)} - n^{(k)} + C]$
2. collect $\psi^{(k+1)}$ and/or extrapolate a new value for $\psi^{(k+1)}$
3. $\nabla\cdot[-q\mu_n n^{(k+1)}\nabla\psi^{(k+1)} + qD_n\nabla n^{(k+1)}] = R(\psi^{(k)}, n^{(k)}, p^{(k)})$
4. collect $\phi_n^{(k+1)}$ and/or extrapolate a new value for $n^{(k+1)}$
5. $\nabla\cdot[q\mu_p p^{(k+1)}\nabla\psi^{(k+1)} + qD_n\nabla p^{(k+1)}] = -R(\psi^{(k)}, n^{(k)}, p^{(k)})$
6. collect $\phi_p^{(k+1)}$ and/or extrapolate a new value for $p^{(k+1)}$
7. $k = k+1$

end

VECTOR EXTRAPOLATION METHODS

Vector extrapolation methods accelerate slowly or non convergent vector sequences. We consider here the Poliynomial Methods. Let $\mathbf{x}_j$, $j = 0, 1, 2\ldots$ be a sequence of N-dimensional column vectors with limit **s**.
A polynomial algorithm finds **s** as a weighted average of $k+1$ terms related with the given sequence, where the k independent weights are found by solving a linear system of size $(N \times k)$. In detail this means:

$$\mathbf{u}_j = \mathbf{x}_{j+1} - \mathbf{x}_j \qquad \mathbf{v}_j = \mathbf{u}_{j+1} - \mathbf{u}_j$$
$$U = [\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{k-1}] \qquad V = [\mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_{k-1}]$$

MPE–Minimal Polynomial Extrapolation:

$$\mathbf{s} = \sum_{j=0}^{k}\gamma_j\mathbf{x}_j \quad \text{with} \quad \gamma_j = \frac{c_j}{\sum_{i=0}^{k} c_i}, \quad j = 0, 1, \ldots, k$$

where the coefficients c_j, $j = 0, \cdots, k$ are obtained solving the linear system $U\mathbf{c} = -\mathbf{u}_k$.

RRE–Reduced Rank Extrapolation:

$$\mathbf{s} = \mathbf{x}_0 + \sum_{i=0}^{k}\xi_i\mathbf{u}_i$$

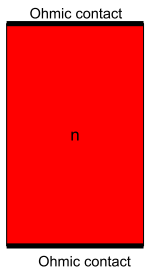
where the coefficients ξ_i, $i = 0, \cdots, k$ are obtained solving the linear system $V\xi = -\mathbf{u}_0$.

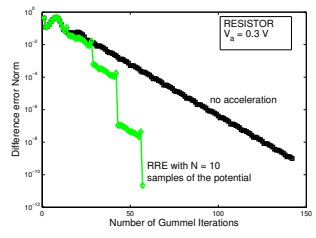
Remarks:

- The explicit knowledge of the sequence generator is not required.
- **s** is find solving a relatively small linear system (i.e., $k << N$).

RESISTOR

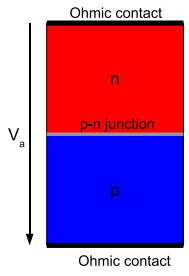
In a resistor the doping C is constant inside the domain and the current density $\vec{j}$ is related to the electric field $\vec{E}$ via the Ohm's law: $\vec{j} = \sigma\vec{E}$ where σ is the conductivity of the material.

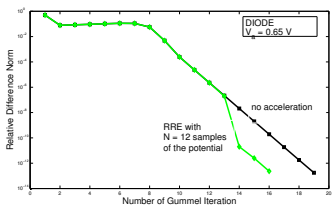




DIODE

Inside a diode the doping function changes sign on surface called p-n junction. Physical processes build up across the p-n junction an electric field. The orientation of this field prevents the free charges (electrons in the n silicon and holes in the p silicon) to cross the junction. However, if we apply a positive external voltage V_a, the electric field is pulled down and the free charges can across the junction. Thus a current flows through the device. Conversely, a negative V_a strengths more the electric field and no free charges can cross the junction. We say that the diode is forward biased in the first case and reverse biased in the second.





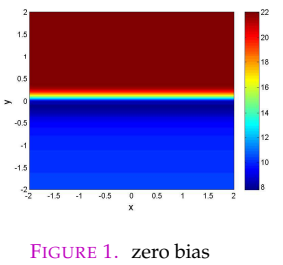


FIGURE 1. zero bias

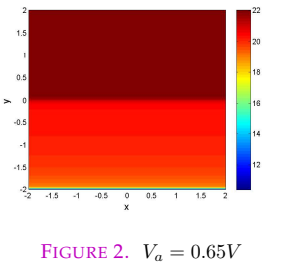
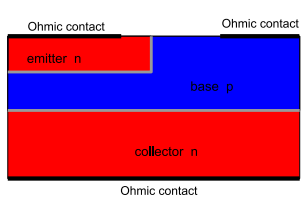
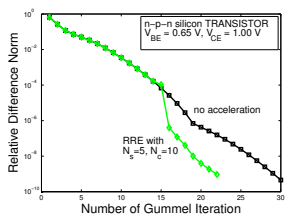


FIGURE 2. $V_a = 0.65V$

TRANSISTOR

A transistor is, basically, formed by two p-n junctions: the emitter-base junction, J_{BE} and the base-collector junction, J_{BC}. Now, consider the case depicted below on the right, assuming that the amount of electrons inside the emitter is some order of magnitude greater then the concentration of holes in the base. In this case J_{BE} is forward biased and J_{BC} is reverse biased. Thus, the electrons in the emitter can cross J_{BE} flowing into the base region. Here, they found the electric field of J_{BC} which, due to it reverse bias, carrying them to the collector region. Thus, a large current flows through the collector contact.





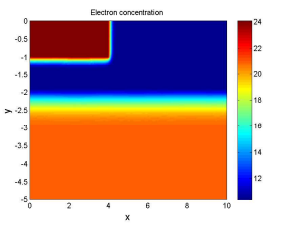


FIGURE 3. zero bias

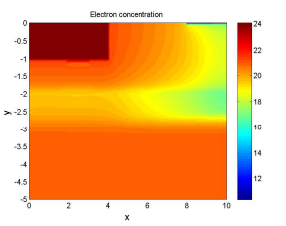
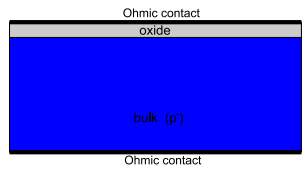


FIGURE 4. $V_{BE} = 0.65V, V_{CE} = 1.00V$

MOS-CAPACITOR

A MOS capacitor is a thin layer of insulator between two, distinct, (semi–) conductor materials. One contact is a conductor sheet made on one face of the insulator. The other contact, made on the opposite face, is a thick semiconductor layer. When an external voltage V_a is applied, an external field appears inside the oxide and attracts electrons just below the silicon-oxide interface. If the external voltage is sufficiently high, an inversion occurs under the oxide, that is the p-silicon becomes n-silicon.



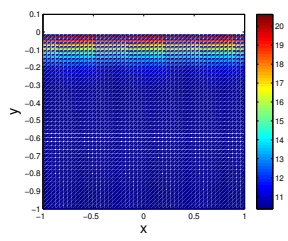


FIGURE 5. zero bias

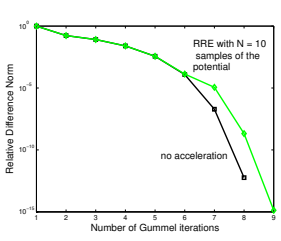
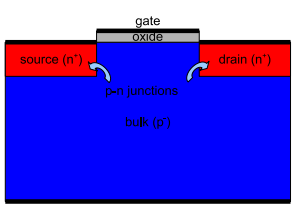


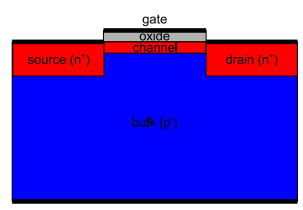
FIGURE 6. $V_a = 2V$

MOSFET

A mosfet is one of the most important electronic devices. It is, basically, the union of a MOS capacitor with two highly doped zones, the source and the drain. Roughly, an n-channel mosfet works as follows. Assume $V_{th} > 0$ be a given threshold voltage.

1. $V_g \leq V_{th}$: the electric field is too low to attract under the oxide a sufficiently large number of source and drain electrons to realize the inversion in the p-silicon region just under the oxide (i.e, the p-silicon becomes an n silicon). Thus, the source and the drain are separated and no current flows throughout the source and drain contacts.
2. $V_g > V_{th}$: the electric field is high enought to create an inversion under the oxide. The drain and the source are joned by the same n-type of semiconductor material (thus, the name n-channel mosfet). A current can flow from source to drain if $V_D - V_S \neq 0$.





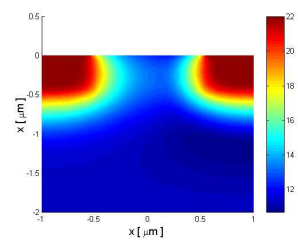


FIGURE 7. $V_g \leq V_{th}$

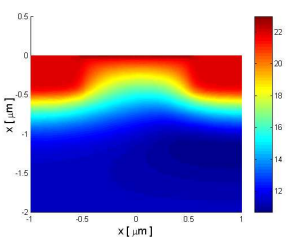
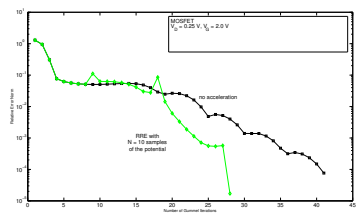


FIGURE 8. $V_g > V_{th}$



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