G. Parmeggiani, 25/11/2019 **Algebra Lineare, a.a. 2019/2020**,

Scuola di Scienze - Corsi di laurea: **Statistica per l'economia e l'impresa**
Statistica per le tecnologie e le scienze

Studenti: **numero di MATRICOLA PARI**

ESERCIZIO TIPO 10

Sia $\mathbf{A}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 1 & \alpha+2i & 0 \\ 2 & 2i & \alpha^2+1 \end{pmatrix}$, dove $\alpha \in \mathbb{C}$.

Per ogni $\alpha \in \mathbb{C}$ si dica qual è $rk(\mathbf{A}_{\boldsymbol{\alpha}})$ e si trovino una base $\boldsymbol{\mathcal{B}}_{\alpha}$ di $C(\mathbf{A}_{\boldsymbol{\alpha}})$ ed una base $\boldsymbol{\mathcal{D}}_{\alpha}$ di $R(\mathbf{A}_{\boldsymbol{\alpha}})$.

$$\mathbf{A}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 1 & \alpha+2i & 0 \\ 2 & 2i & \alpha^2+1 \end{pmatrix} \xrightarrow{E_{31}(-2)E_{21}(-1)} \begin{pmatrix} 1 & i & 0 \\ 0 & \alpha+i & 0 \\ 0 & 0 & \alpha^2+1 \end{pmatrix} = \mathbf{B}_{\boldsymbol{\alpha}}$$

$\boxed{1^o CASO}$ $\alpha = -i$:

$$\mathbf{B}_{-\boldsymbol{i}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{U}_{-\boldsymbol{i}}$$

$$rk(\mathbf{A}_{-\boldsymbol{i}}) = 1, \quad \boldsymbol{\mathcal{D}}_{-i} = \left\{ \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \right\}, \quad \boldsymbol{\mathcal{B}}_{-i} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$$

$\boxed{2^o CASO}$ $\alpha \neq -i$

$$\mathbf{B}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 0 & \alpha+i & 0 \\ 0 & 0 & \alpha^2+1 \end{pmatrix} \xrightarrow{E_3(\frac{1}{\alpha+i})E_2(\frac{1}{\alpha+i})} \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha-i \end{pmatrix} = \mathbf{C}_{\boldsymbol{\alpha}}$$

$\boxed{1^o Sottocaso}$ $\quad \alpha = i :$

$$\mathbf{C}_{\boldsymbol{i}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{U}_{\boldsymbol{i}}$$

$$rk(\mathbf{A}_{\boldsymbol{i}}) = 2, \quad \boldsymbol{\mathcal{D}}_{\ i} = \left\{ \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}, \quad \boldsymbol{\mathcal{B}}_{\ i} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}; \begin{pmatrix} i \\ 3i \\ 2i \end{pmatrix} \right\}$$

$\boxed{2^o Sottocaso}$ $\quad \alpha \neq -i, i :$

$$\mathbf{C}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha - i \end{pmatrix} \xrightarrow{E_3(\frac{1}{\alpha - i})} \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{U}_{\boldsymbol{\alpha}}$$

$$rk(\mathbf{A}_{\boldsymbol{\alpha}}) = 3,$$

$$\boldsymbol{\mathcal{D}}_{\ \alpha} = \left\{ \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$\boldsymbol{\mathcal{B}}_{\ \alpha} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}; \begin{pmatrix} i \\ \alpha + 2i \\ 2i \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ \alpha^2 + 1 \end{pmatrix} \right\}.$$

N.B.: Essendo in questo caso $C(\mathbf{A}_{\boldsymbol{\alpha}}) \leq \mathbb{C}^3$ e $\dim(C(\mathbf{A}_{\boldsymbol{\alpha}})) = 3 = \dim(\mathbb{C}^3)$, allora $C(\mathbf{A}_{\boldsymbol{\alpha}}) = \mathbb{C}^3$ e si sarebbe potuto prendere $\boldsymbol{\mathcal{B}}_{\ \alpha} = \{\mathbf{e_1}; \mathbf{e_2}; \mathbf{e_3}\}$.

N.B.: Essendo in questo caso $R(\mathbf{A}_{\boldsymbol{\alpha}}) \leq \mathbb{C}^3$ e $\dim(R(\mathbf{A}_{\boldsymbol{\alpha}})) = 3 = \dim(\mathbb{C}^3)$, allora $R(\mathbf{A}_{\boldsymbol{\alpha}}) = \mathbb{C}^3$ e si sarebbe potuto prendere $\boldsymbol{\mathcal{D}}_{\ \alpha} = \{\mathbf{e_1}; \mathbf{e_2}; \mathbf{e_3}\}$.