

Svolgimento degli Esercizi per casa 2 (prima parte)

[1] Siano $\mathbf{A} = \begin{pmatrix} 2-3i & 1+i \\ 0 & i \\ 1-i & 1 \end{pmatrix}$, $\mathbf{B} = (2 \ 1+i)$, $\mathbf{C} = \begin{pmatrix} 3+5i \\ 6 \\ 2-2i \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 7+i & 2+3i \\ 3-2i & 0 \end{pmatrix}$.

(a) Di ciascuna delle precedenti matrici si calcolino la trasposta, la coniugata e la H-trasposta.

(b) Si calcoli $(\mathbf{A}^H \overline{\mathbf{C}} + i\mathbf{B}^T) \overline{\mathbf{B}} + (1+3i)\mathbf{D}^H$.

$$\begin{aligned} \mathbf{A}^T &= \begin{pmatrix} 2-3i & 0 & 1-i \\ 1+i & i & 1 \end{pmatrix} & \overline{\mathbf{A}} &= \begin{pmatrix} 2+3i & 1-i \\ 0 & -i \\ 1+i & 1 \end{pmatrix} & \mathbf{A}^H &= \begin{pmatrix} 2+3i & 0 & 1+i \\ 1-i & -i & 1 \end{pmatrix} \\ \mathbf{B}^T &= \begin{pmatrix} 2 \\ 1+i \end{pmatrix} & \overline{\mathbf{B}} &= (2 \ 1-i) & \mathbf{B}^H &= \begin{pmatrix} 2 \\ 1-i \end{pmatrix} \\ \mathbf{C}^T &= (3+5i \ 6 \ 2-2i) & \overline{\mathbf{C}} &= \begin{pmatrix} 3-5i \\ 6 \\ 2+2i \end{pmatrix} & \mathbf{C}^H &= (3-5i \ 6 \ 2+2i) \\ \mathbf{D}^T &= \begin{pmatrix} 7+i & 3-2i \\ 2+3i & 0 \end{pmatrix} & \overline{\mathbf{D}} &= \begin{pmatrix} 7-i & 2-3i \\ 3+2i & 0 \end{pmatrix} & \mathbf{D}^H &= \begin{pmatrix} 7-i & 3+2i \\ 2-3i & 0 \end{pmatrix} \end{aligned}$$

$$(\mathbf{A}^H \overline{\mathbf{C}} + i\mathbf{B}^T) \overline{\mathbf{B}} + (1+3i)\mathbf{D}^H =$$

$$= (\begin{pmatrix} 2+3i & 0 & 1+i \\ 1-i & -i & 1 \end{pmatrix} \begin{pmatrix} 3-5i \\ 6 \\ 2+2i \end{pmatrix} + i \begin{pmatrix} 2 \\ 1+i \end{pmatrix}) (2 \ 1-i) + (1+3i) \begin{pmatrix} 7-i & 3+2i \\ 2-3i & 0 \end{pmatrix} =$$

$$\begin{aligned}
&= \left(\begin{pmatrix} (2+3i)(3-5i) + (1+i)(2+2i) \\ (1-i)(3-5i) - 6i + 2 + 2i \end{pmatrix} + \begin{pmatrix} 2i \\ i(1+i) \end{pmatrix} \right) (2 - 1 - i) + \\
&\quad + \begin{pmatrix} (1+3i)(7-i) & (1+3i)(3+2i) \\ (1+3i)(2-3i) & 0 \end{pmatrix} = \\
&= \left(\begin{pmatrix} 6+9i-10i+15+2+2i+2i-2 \\ 3-3i-5i-5-6i+2+2i \end{pmatrix} + \begin{pmatrix} 2 \\ -1+i \end{pmatrix} \right) (2 - 1 - i) + \\
&\quad + \begin{pmatrix} 7+21i-i+3 & 3+9i+2i-6 \\ 2+6i-3i+9 & 0 \end{pmatrix} = \\
&= \left(\begin{pmatrix} 21+3i \\ -12i \end{pmatrix} + \begin{pmatrix} 2i \\ -1+i \end{pmatrix} \right) (2 - 1 - i) + \begin{pmatrix} 10+20i & -3+11i \\ 11+3i & 0 \end{pmatrix} = \\
&= \begin{pmatrix} 21+5i \\ -1-11i \end{pmatrix} (2 - 1 - i) + \begin{pmatrix} 10+20i & -3+11i \\ 11+3i & 0 \end{pmatrix} = \\
&= \begin{pmatrix} 2(21+5i) & (21+5i)(1-i) \\ 2(-1-11i) & (-1-11i)(1-i) \end{pmatrix} + \begin{pmatrix} 10+20i & -3+11i \\ 11+3i & 0 \end{pmatrix} = \\
&= \begin{pmatrix} 42+10i & 21+5i-21i+5 \\ -2-22i & -1-11i+i-11 \end{pmatrix} + \begin{pmatrix} 10+20i & -3+11i \\ 11+3i & 0 \end{pmatrix} = \\
&= \begin{pmatrix} 42+10i & 26-16i \\ -2-22i & -12-10i \end{pmatrix} + \begin{pmatrix} 10+20i & -3+11i \\ 11+3i & 0 \end{pmatrix} = \begin{pmatrix} 52+30i & 23-5i \\ 9-19i & -12-10i \end{pmatrix}
\end{aligned}$$