

HOMWORK 1

SOLUTIONS DUE BY: OCTOBER 14TH.

Exercise 1. Compute

$$\lim_{n \rightarrow +\infty} n \int_0^{+\infty} \log \left(1 + \frac{e^{-x}}{n} \right) dx.$$

Exercise 2. Let

$$F(x) = \int_0^{+\infty} \frac{1 - e^{-xt^2}}{t^2} dt.$$

- i) Determine the set of $x \in \mathbb{R}$ such that $F(x)$ is well defined (domain of F).
- ii) Discuss carefully the derivability of F on its domain and compute F' . State the general theorems you need.
- iii) Use the previous results to determine explicitly F .

Exercise 3. Let

$$f(x, y) := \frac{1}{1 - xy}, \text{ a.e. } (x, y) \in [0, 1]^2$$

Recall that $\frac{1}{1-q} = \sum_{n=0}^{\infty} q^n$ for every $|q| < 1$. Use this to discuss if $f \in L^1([0, 1]^2)$.