FUNDAMENTALS OF MATHEMATICAL ANALYSIS II — ENSTP

EXAM SIMULATION (SOLUTION BY WED 17TH OF JUNE)

Exercise 1. Solve the Cauchy problem

$$\begin{cases} y' = y^2 - y - 1, \\ \\ y(0) = 1/2. \end{cases}$$

Exercise 2. Let

$$f(x, y) := 3(x^2 + y^2)^2 - 4xy^3.$$

i) Compute, if it exists, $\lim_{(x,y)\to\infty_2} f(x, y)$.
ii) Determine min/max for f on $\mathbb{R}^2$ (if any).
iii) Determine min/max of f on the disk $\{x^2 + y^2 \leqslant 1\}$ (if any).

Exercise 3. Determine min/max of $f(x, y, z) = xyz$ on the set

$$D := \{(x, y, z)^3 \ : \ 2x^2 + y^2 + z^2 = 1\}.$$

Exercise 4. Let

$$D := \{(x, y, z) \in \mathbb{R}^3 \ \ x^2 + y^2 \leqslant z \leqslant 1 - (x + y)\}.$$

i) Draw D.
ii) Determine volume of D.

Time: 4h.