FUNDAMENTALS OF MATHEMATICAL ANALYSIS 2 MENG — PROF. PAOLO GUIOTTO

2020 RESIT EXAMS SIMULATION

Exercise 1. Consider

$$\mathscr{M} := \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z^2, \quad y^2 + z^2 = x + 1\}.$$

i) Is $\mathscr{M}$ compact? Justify your answer.
ii) Show that $g(x,y,z) = (x^2 + y^2 - z^2, x + 1 - y^2 - z^2)$ is submersive on $\mathscr{M}$.
iii) Determine all the points of $\mathscr{M}$ at min/max distance to the origin (if any).

Exercise 2. Consider the system

$$\begin{cases} x' = y, \\ \\ y' = \sinh x. \end{cases}$$

Accept local existence and uniqueness.

i) Determine the stationary solutions.
ii) Determine a non constant prime integral.
iii) Plot the phase portrait of the system. Are there global solutions? Are there periodic solutions?
iv) Solve the Cauchy problem $x(0) = 0$, $y(0) = 2$.

Exercise 3. Let $\Omega := \{(x,y,z) \in \mathbb{R}^3 \ : \ \sqrt{(x^2 + y^2)^3} \leqslant z \leqslant 1\}$.

i) Compute the area of $\partial\Omega$.
ii) Compute the outward flux of $\vec{F} = (y, x, z^2)$ from Ω, determining also its component on $\mathscr{M} := \Omega \cap \{z = \sqrt{(x^2 + y^2)^3}\}$.

Exercise 4. Consider the equation

$$y' = -\frac{1}{\log(ty)}.$$

i) Determine the domain D of local existence and uniqueness, constant solutions (if any) and regions of D where solutions are increasing/decreasing.
ii) Show that if $y(t)$ is a solution for the equation then $-y(-t)$ is a solution.

Let $y :]\alpha, \beta[\longrightarrow \mathbb{R}$ be the solution of the Cauchy problem $y(1/2) = 1$.

iii) Show that y is monotone and deduce that $\beta < +\infty$.
iv) Determine the concavity of y. What can you say about α?
v) Plot a graph of the solution.

Time: 4h