A numerical method for bisingular Cauchy integral equations

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This talk deals with the numerical solution of bisingular Cauchy integral equations of the first kind, defined on the square $S = [-1, 1] \times [-1, 1]$, having the following form

$$(D+K)f = g$$

where f is the bivariate unknown function, g is a given right-hand side, D is the dominant operator

$$Df(t,s) = \frac{1}{\pi^2} \oint_S \frac{f(x,y)}{(x-t)(y-s)} \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dx \, dy,$$

and K is the perturbation operator

$$Kf(t,s) = \int_{S} k(x,y,t,s) f(x,y) \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dx \, dy$$

with k a given kernel function.

For its solution we propose a numerical method based on a polynomial approximation of the unknown function f. We examine the stability of the proposed method, discuss the convergence, and analyze the conditioning of the linear system we solve. Moreover, we illustrate numerical tests showing the efficiency of the approach.