Extrapolation Methods for fixed point Multilinear PageRank computations

Stefano Cipolla[1], Michela Redivo-Zaglia[2] and Francesco Tudisco[3]

[1] University of Padua, Dept. of Mathematics, Padua, Italy. cipolla@math.unipd.it
[2] University of Padua, Dept. of Mathematics, Padua, Italy. michela@math.unipd.it
[3] University of Strathclyde, Dept. of Mathematics and Statistics, Glasgow, UK. f.tudisco@strath.ac.uk

A Markov Chain is a discrete stochastic process $\{X_t\}_{t=0}^{\infty}$ over a finite state space where the probability distribution of X_{t+1} depends on the previous $X_t, \ldots, X_0$. However, the classic "Markov property" specifies that the transition probability to the next state only depends on the probability of the current state, i.e. $\mathbb{P}(X_{t+1}|X_t, \ldots, X_0) = \mathbb{P}(X_{t+1}|X_t)$. Nevertheless, there are situations where it is important to keep track of what happens further in the past, leading to what we call *Higher Order Markov Chain*.

Given a random walk on a directed graph, the PageRank modification [1] builds a new Markov chain that always has a unique stationary distribution. Recently this idea has been extended to Higher Order Markov Chains [2]. Although this extension has attractive theoretical properties, it is computationally intractable for problems of large size; hence an approximation of the ideal Higher Order PageRank vector is introduced, called Multilinear PageRank. The Multilinear PageRank vector can be interpreted as the stationary distribution of a non-Markovian stochastic process called the "spacey random surfer".

In this talk, after a short survey on results about the existence/uniqueness of the solution and on the state-of-the-art of computational techniques for the Multilinear PageRank vector, we will show how its computation can be considerably sped-up using extrapolation techniques. In particular we will show how the sequence generated by two fixed point-type techniques as the SS-HOPM [3] and the Inner-Outer Method [2], are accelerated using the The Simplified Topological ϵ-Algorithm (STEA) [4] in the restarted form [5]. The considerable improvement of the rate of convergence in the accelerated version, obtained at the cost of a fixed number of scalar products per step, suggests that the sequences generated by the considered methods are particularly close to the Shanks Kernel and hence encourages further theoretical investigation.

References

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