Exact Formulae and Matrix–less eigensolvers for Block Banded Toeplitz-like matrices

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Bogoya, Böttcher, Grudsky, and Maximenko have recently obtained the precise asymptotic expansion for the eigenvalues of a sequence of Toeplitz matrices $\{T_n(f)\}_n$, under suitable assumptions on the associated generating function f [2]. An evident restriction is that f has to be polynomial, monotone and scalar-valued.

In this talk we focus on the case of f being a $s \times s$ matrix-valued trigonometric polynomial, $s \geq 1$, and $\{T_n(f)\}_n$ a sequence of block Toeplitz matrix generated by f, with size $N(n, s) = sn$, where the case $s = 1$ corresponds to that already treated in the literature [6].

Following the proposal in the scalar-valued case, we devise an extrapolation algorithm [1, 4, 5, 6] (see also [3]) for computing the eigenvalues in the present setting regarding banded symmetric block Toeplitz matrices, with a high level of accuracy and with a low computational cost.

We use the asymptotic expansion to study the spectral properties of special block Toeplitz structures and we show exact formulae for the eigenvalues of the stiffness matrices coming from the Q_p Lagrangian FEM approximation of a second order elliptic differential problem [7]. Numerical results are presented and critically discussed.

References

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