Factoring Fiedler pencils and their distance from orthogonal-plus-low-rank

Gianna Del Corso [1], Federico Poloni[1], Leonardo Robol[2] and Raf Vandebril[3]

[1] *University of Pisa.* `federico.poloni, giannna.delcorso@unipi.it`
[2] *ISTI-CNR.* `leonardo.robol@isti.cnr.it.` *KU Leuven.* `raf.vandebril@cs.kuleuven.be`

In [2], the authors obtain a backward stable algorithm for computing the eigenvalues of a matrix polynomial $P(z) = z^d + A_{d-1}z^{d-1} + \ldots + A_1 z + A_0 \in \mathbb{C}[z]^{k\times k}$ using a fast eigensolver on the classical Frobenius (column-based) companion matrix. The two main properties that make it possible are:

1. this companion matrix can be factored into the product of k analogous companion matrices of *scalar* polynomials ($k = 1$);
2. this companion matrix is a small-rank modification of an orthogonal matrix.

We show that both these results hold also for a larger class of companion matrices introduced by Fiedler [1, 3]. The matrices in this class can be obtained as products of elementary matrices of the form

$$\begin{bmatrix} 0 & I_k \\ I_k & A \end{bmatrix}, \quad A \in \mathbb{C}^{k\times k}, \tag{1}$$

suitably padded with identities.

To obtain the first result, the main ingredient is extending the flow graph notation for Fiedler matrices introduced in [4] with the novel idea of 'breaking up' a block elementary Fiedler matrix (1) into the product of several scalar ones (i.e., with the same structure but $k = 1$). The decompositions arising from this factorization are easy to visualize graphically.

The second result also stems from factoring Fiedler pencils and rearranging the various terms, and leads to a more general result that on when a matrix is orthogonal-plus-low-rank.

References

[1] E. N. Antoniou, S. Vologiannidis, *A new family of companion forms of polynomial matrices*, Electron. J. Linear Algebra, 11(2004) 78–87.

[2] J. L. Aurentz, T. Mach, L. Robol, R. Vandebril, D. S. Watkins, *Fast and backward stable computation of the eigenvalues of matrix polynomials*, Math. Comput., in press.

[3] M. Fiedler *A note on companion matrices*, Linear Algebra Appl., 372(2003) 325–331.

[4] F. Poloni,G. M. Del Corso, *Counting Fiedler pencils with repetitions*, Linear Algebra and its Applications, 532(2017) 463–499.