Perturbations of Hermitian Matrices and Applications to Spectral Symbols

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It is often observed in practice that matrix sequences $\{A_n\}_n$ generated by discretization methods applied to linear differential equations, possess a *Spectral Symbol*, that is a measurable function describing the asymptotic distribution of the eigenvalues of A_n. Sequences composed by Hermitian matrices own real spectral symbols, that can be derived through the axioms of *Generalized Locally Toeplitz* sequences [1].

The spectral analysis of matrix-sequences which can be written as a non-Hermitian perturbation of a given Hermitian matrix-sequence has been performed in a previous work by Leonid Golinskii and the second author [2]. A result was proven but under the technical restrictive assumption that the involved matrix-sequences are uniformly bounded in spectral norm. Nevertheless that result had a remarkable impact in the analysis of spectral distribution and clustering of matrix-sequences coming from various applications, mainly in the context of the numerical approximation of partial differential equations (PDEs) and related preconditioned matrix-sequences.

In this presentation, we propose a new result that does not require the boundedness of the sequences and permits to enlarge substantially the class of problems, such as variable-coefficient PDEs and preconditioned matrix-sequences with unbounded coefficients.

References

[1] Garoni C., Serra-Capizzano S. *Generalized Locally Toeplitz Sequences: Theory and Applications (Volume I).* Springer (2017).

[2] Golinskii L., Serra-Capizzano S. *The asymptotic properties of the spectrum of nonsymmetrically perturbed Jacobi matrix sequences.* J. Approx. Theory 144 (2007) 84–102.