Designing constraint-preconditioned Krylov methods for the solution of regularized saddle-point systems

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We are interested in the iterative solution of regularized saddle-point systems where the leading block of the matrix can be either symmetric or non-symmetric. These systems arise in many areas of scientific computing, such as interior point and augmented Lagrangian methods for constrained optimization, and stabilized finite-element discretizations of incompressible flow problems [1, 2].

When the leading block is symmetric and satisfies additional conditions, e.g., accounting for the local convexity of an associated minimization problem, the system can be solved by using the conjugate gradient method coupled with a constraint preconditioner, a choice that has proved to be very effective, especially in optimization applications. In this work, we consider more general leading blocks and investigate the design of constraint-preconditioned variants of other Krylov methods, by focusing on the underlying basis-generation processes.

We build upon [3] to provide general guidelines that allow us to specialize any Krylov method to regularized saddle-point systems. In particular, we obtain constraint-preconditioned variants of Lanczos and Arnoldi-based methods, including MINRES, SYMMLQ, GMRES(m) and DQGMRES. A numerical illustration of their behaviour is provided, using systems arising in constrained optimization and fluid-flow simulation.

References

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