

# Iterative Construction of Non-Symmetric Factored Sparse Approximate Inverse Preconditioners

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Factored sparse approximate inverses (FSAI) play a key-role in the efficient algebraic preconditioning of sparse linear systems of equations. For SPD problems remarkable results are obtained by building the FSAI non-zero pattern iteratively during its computation [1]. Unfortunately, an equivalent algorithm still is missing in the non-symmetric case. In the present contribution we explore the possibility of iteratively computing FSAI for non-symmetric matrices by using an incomplete Krylov subspace bi-orthogonalization procedure. Another adaptive technique relies on the idea of directly minimizing the two norm of the off-diagonal row(/column) of the preconditioned matrix. Finally, as reference algorithm, a factorized sparse approximate inverse on static pattern is considered.

The main idea behind these approaches is to build two real sparse triangular factors ( $W$  is lower triangular and  $Z$  is upper triangular) such that:

$$WAZ = D \tag{1}$$

where  $A \in \mathbb{R}^{n \times n}$  is the original non-symmetric matrix and  $D$  is the preconditioned matrix. Factors  $W$  and  $Z$  should be sparse, cheap to compute and effective, i.e.  $D$  tends to be diagonal.

The three mentioned algorithms are intrinsically parallel as they compute the approximate inverse row(/column)-wisely, with each row independently computed from the others. In this preliminary work, we show the effectiveness of the bi-orthogonalization based preconditioner for Krylov subspace iterative methods, like BiCGstab and GMRES. We compare this approach also with the norm minimization technique and with the computation relying on a static pattern.

## References

- [1] C. Janna, M. Ferronato, F. Sartoretto, G. Gambolati *FSAIPACK: A software package for high-performance factored sparse approximate inverse preconditioning*, ACM Transactions on Mathematical Software (TOMS), 42 (2015) 10:1-10:26.