Fast Computation of Toeplitz Eigenvalues through Asymptotic Expansions and Extrapolation

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Extrapolation is known to be one of the most successful ways for accelerating the convergence of numerical methods [2]. In the words of Birkhoff and Rota, "its usefulness for practical computations can hardly be overestimated". In the presence of an asymptotic expansion for the quantity to be approximated, a "canonical" extrapolation method arises; think, for example, to Romberg's integration method, which arises from the Euler–Maclaurin expansion associated with the trapezoidal formula.

In this presentation, we discuss a recently conjectured asymptotic expansion for the eigenvalues of banded symmetric Toeplitz matrices [5]. We also describe the related extrapolation method, which allows the fast computation of the spectrum of such matrices [4, 5]. Further applications of this method include the fast computation of the eigenvalues of both preconditioned banded symmetric Toeplitz matrices [1, 4] and PDE discretization matrices [3].

References

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