A Schur algorithm for rational matrix equations

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Let r be a rational function. We consider the matrix equation $r(X) = A$, where A and X are a given and an unknown square complex matrix, respectively, and $r(X)$ should be understood in the sense of functions of matrices [1].

After a brief classification of the solutions, we describe an algorithm, based on a recursion on the Schur normal form of A, to compute all the *well-posed* solutions of the aforementioned equation. The algorithm is constructed in a fashion similar to existing algorithms for specific problems, such as the ones for the equation $X^p = A$ [2], and behaves in a similar, stable way. Moreover, in the case of real data, it is able to compute the real solutions using only real arithmetic.

As an application, we propose a new algorithm for computing the matrix logarithm, built on the inverse scaling and squaring method [3], but relying on a different rational approximation.

References

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