A novel AMG approach based on adaptive smoothing and prolongation for ill-conditioned linear systems

Carlo Janna [1], Victor A. Paludetto Magri[2] and Andrea Franceschini [3]

[1] *University of Padova and M3E s.r.l..* carlo.janna@unipd.it
[2] *University of Padova.* victor.magri@dicea.unipd.it
[3] *University of Padova.* franc90@dmsa.unipd.it

The numerical simulation of modern engineering problems can easily incorporate millions or even billions degrees of freedom. In several applications, these simulations require the solution to sparse linear systems of equations, and algebraic multigrid (AMG) methods are often standard choices as iterative solvers or preconditioners [1]. This happens due to their high convergence speed guaranteed even in large size problems, which is a consequence of the AMG ability of reducing particular error components across their multilevel hierarchy. Despite carrying the name "algebraic", most of these methods still rely on additional information other than the global assembled sparse matrix, as for instance the knowledge of the operator near kernel. This fact somewhat limits their applicability as black-box solvers. In this work, we introduce a novel AMG approach featuring the adaptive Factored Sparse Approximate Inverse (aFSAI) [2] method as a flexible smoother as well as three new approaches to adaptively compute the prolongation operator. We assess the performance of the proposed AMG through the solution of a set of model problems along with real-world engineering test cases. Moreover, comparisons are made with the aFSAI and BoomerAMG preconditioners, showing that our new method proves to be superior to the first strategy and comparable to the second one, if not better as in the elasticity problems.

References

[1] J. Xu and L. Zikatanov, *Algebraic multigrid methods*, Acta Numerica, 26 (2017), pp. 591–721.

[2] C. Janna, M. Ferronato, F. Sartoretto, and G. Gambolati, FSAIPACK: A software package for high-performance factored sparse approximate inverse preconditioning, ACM Trans. Math. Softw., 41 (2015), pp. 10:1–10:26.