Low-rank updates and a divide-and-conquer method for linear matrix equations

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In this work we study how the solutions of certain linear matrix equations behave when the original coefficients are modified with low-rank perturbations. More precisely, given the solution X_0 of the Sylvester equation $AX_0 + X_0B = C$, and 3 low-rank matrices $\delta A, \delta B$ and δC, we are interested in characterizing the update δX that verifies

$$(A + \delta A)(X_0 + \delta X) + (X_0 + \delta X)(B + \delta B) = C + \delta C.$$

Under reasonable assumptions, δX turns out to have a low numerical rank and allows to be efficiently approximated by means of Krylov subspace techniques. We show how to exploit this property to design divide and conquer methods for solving large-scale Sylvester equations whose coefficients are represented in the HODLR and HSS formats. This comprises the case of banded and quasiseparable coefficients.