Numerical linear algebra techniques for efficient RBFs interpolation and collocation

S. De Marchi[1], A. Martínez[1] and E. Perracchione[1]

[1] *Dip. Matematica "Tullio Levi-Civita", Università degli studi di Padova.*
{demarchi, acalomar, emma.perracchione}@math.unipd.it

Scattered data interpolation using Radial Basis Functions (RBFs) involves solving ill-conditioned Symmetric Positive Definite (SPD) linear systems; refer e.g. to [6] for further details. We will discuss the properties (conditioning, density) of the interpolation matrices for both global and compactly supported kernels, depending on the value of the shape parameter for both classical global interpolation and local methods based on Partition of Unity (PU). The severe ill-conditioning of the interpolation matrices causes theoretically SPD matrices to be not numerically SPD. We will discuss the benefits provided by Tikhonov regularization techniques to guarantee the stability of the solution, as well as preconditioned iterative methods for the solution by collocation of elliptic boundary value problems [2].

Also efficient numerical linear algebra tools are needed in the computation of rational RBF interpolants [4, 5]. Rational RBF interpolation reveals particularly suitable for approximating functions that display oscillations or steep gradients. The study described in [3] reveals that the method is robust enough to accurately fit data coming from applications, such as Earth's topography. Moreover, when compactly supported RBFs are used, it enables us to increase the sparsity of the kernel matrices and at the same time to maintain a good accuracy. Furthermore, since a global interpolation method cannot handle truly large sets of points, an efficient implementation via the PU method is carried out. The resulting scheme requires the solution of a set of generalized eigenvalue problems, one for each local subdomain. We will describe an efficient algorithm for solving these local eigenvalue problems by means of the combination of the power method and the Deflation-Accelerated Conjugate Gradient (DACG) method [1]. We will present results showing that with this efficient implementation the proposed method outperforms the classical and rescaled PU schemes.

References

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