Matrix-free iterative solvers for isogeometric analysis

Giancarlo Sangalli[1] and Mattia Tani[2]

[1] *Università di Pavia.* giancarlo.sangalli@unipv.it
[2] *Università di Pavia.* mattia.tani@unipv.it

Isogeometric analysis (IGA) is a method to numerically solve partial differential equations (PDEs). It is based on the idea of using B-splines (and their generalizations) both for the parametrization of the domain, as it is typically done by computer aided design software, and for the representation of the unknown solution. One interesting feature of IGA is the possibility of using high-degree high-regularity splines (the so-called $k-$refinement) as they deliver higher accuracy per degree-of-freedom in comparison to C^0 finite elements [1].

The computational cost of a solver for a linear PDE problem is the sum of the cost of the formation of the system matrix and the cost of the solution of the linear system. Unfortunately, it is known if these two steps are performed using the approaches that are standard in the context of C^0 finite elements, their computational cost increases dramatically with the spline degree. This makes the k-refinement unfeasible for practical problem, where quadratic or cubic splines are typically preferred.

Several improvements have been achieved recently. In [2], the authors discuss a preconditioner for scalar elliptic problems, based on an old idea, which is robust with respect to both the mesh size h and the spline degree p. Moreover, in [3] a novel method is developed that allows the formation of the stiffness matrix with almost optimal complexity. In the recent work [4], these two approaches are combined with a third ingredient: a matrix-free implementation.

In this talk we discuss the overall strategy, which is very beneficial in terms of both memory and computational cost. In particular, we show that memory required is practically independent of p and that the cost depends on p only mildly. The numerical experiments show that, with the new implementation, the $k-$refinement becomes appealing from a computational point of view. Indeed, increasing the degree and continuity leads to orders of magnitude higher computational efficiency.

References

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