Complex Analysis
Academic year 2012-2013
Detailed syllabus

Suggested references:
- slides of the lectures given in class;
- Giuseppe De Marco, Selected Topics of Complex Analysis, (2012);
- Theodore W. Gamelin, Complex Analysis. UTM, Springer-Verlag (2001);
- Robert B. Ash, W. P. Novinger, Chapter 7 of Complex Variables. Second Edition, Dover Books on Mathematics (2007).

All arguments are intended with proofs (done at class or left as exercise).

- Conformal maps on the complex plane, relation with holomorphic functions. Local injectivity and (local) biholomorphisms. Examples of groups of biholomorphisms. Moebius transformations.

- Maximum modulus theorem. Schwarz's lemma. Automorphisms of the unit disk. Local normal form.

- Riemann Mapping theorem. Domains with the Square Root Property. Topology of convergence on compact subsets. Montel's and Hurwitz's theorems.

- Vitali's theorm. Holomorphic functions defined by integrals: proper and improper case.

- The Gamma and Beta functions. Wielandt's uniqueness theorem. Euler's supplement. Multiplication formula.

- Series of holomorphic functions. Examples (the sinus and the cotangent series).

- The Stirling formula. Gudermann's series.

- Hankel's integral representation.

- Infinite products, infinite products of holomorphic functions. Gauss' product formula. Weierstrass' factors. Weierstrass' product theorem. Factorization theorem.

- Divisors and meromorphic functions on the plane and on the extended plane. The Picard group. Weierstrass' product theorem for domains.

- Jensen's formula and Jensen's inequality. Blaschke products.

- Partial Fractions Decompositions, Mittag-Leffler's theorem.

- Ideal theory for the ring of holomorphic functions: great common divisors, finitely generated and prime ideals, closed ideals, Wedderburn's lemma.

- Approximation by rational functions. 1-chains and 1-cycles. Cauchy's formula for compact sets. Bounded and unbounded connected components. Pole shifting lemma. Runge's theorem.

- Characterization of simply connected regions.

- Riemann's Zeta function. Bernoulli's numbers and Euler's identities. Euler's product formula. Integral representation of the Zeta function. Riemann's relation. Zeros of Zeta and the critical strip.

- The Prime Number Theorem. Chebyschev's function. The Laplace and Mellin transform.