

COMPLEX ANALYSIS

Academic year 2013-2014

Detailed syllabus

Suggested references:

- slides of the lectures given in class;
- Giuseppe De Marco, Selected Topics of Complex Analysis, (2012);
- Walter Rudin - Real and Complex Analysis, Third edition. McGraw-Hill (1986);
- Theodore W. Gamelin, Complex Analysis. UTM, Springer-Verlag (2001);
- Robert B. Ash, W. P. Novinger, Chapter 7 of Complex Variables. Second Edition, Dover Books on Mathematics (2007).

Whenever not specified, all arguments are intended with proofs, with the exclusion of those left as an exercise.

- Conformal maps on the complex plane. Local injectivity and (local) biholomorphisms. Examples of groups of biholomorphisms. Moebius transformations, automorphisms of the extended plane.
- Maximum modulus theorem. Schwarz's lemma. Automorphisms of the unit disk. Local normal form.
- Riemann Mapping theorem. Domains with the Square Root Property. Topology of convergence on compact subsets. Montel's theorem and Hurwitz's theorem (with proof).
- Vitali's theorem. Holomorphic functions defined by integrals: proper and improper case.
- The Gamma and Beta functions. Wielandt's uniqueness theorem. Euler's supplement. Multiplication formula.
- Series of holomorphic functions.
- Hankel's integral representation (without proof).
- Gudermann's series. The Stirling formula.
- Infinite products, infinite products of holomorphic functions. Gauss' product formula.
- Approximation by rational functions. 1-chains and 1-cycles. Cauchy's formula for compact sets. Bounded and unbounded connected components. Pole shifting lemma. Runge's theorem for compact subsets and for open subsets.
- Characterization of simply connected regions.
- Principal parts distributions, Mittag-Leffler's theorem, partial fractions decompositions. The sine and the cotangent series.
- Divisors and meromorphic functions on the plane and on the extended plane. The Picard group. Weierstrass' product and factorization theorems.
- Jensen's formula and Jensen's inequality. Blaschke products.

- Ideal theory for the ring of holomorphic functions: great common divisors, Wedderburn's lemma, finitely generated and prime ideals, closed ideals.
 - Riemann's Zeta function. Bernoulli's numbers and Euler's identities. Euler's product formula. Integral representation of the Zeta function. Riemann's relation. Zeros of the Zeta and the critical strip.
 - The Prime Number Theorem. Chebyshev's function (without the proof of Lemma 1). The Laplace and Mellin transform.
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