Complex Analysis

Academic year 2016-2017

Detailed syllabus

Suggested references:
- slides of the lectures given in class;
- J.-P. Schneiders, *Fonctions de Variables Complexes*. Université de Liège (2010);
- G. De Marco, *Selected Topics of Complex Analysis*. Università di Padova (2012);
- T. W. Gamelin, *Complex Analysis*. UTM, Springer-Verlag (2001);
- W. Rudin, *Real and Complex Analysis, Third edition*. McGraw-Hill (1986);
- R. B. Ash, W. P. Novinger, Chapter 7 of *Complex Variables. Second Edition*. Dover (2007).

Whenever not specified, all arguments are intended with proofs, with the exclusion of the omitted ones.

- Cauchy's argument principle. Rouché's and Hurwitz's theorems.
- Open mapping theorem. Local injectivity and (local) biholomorphisms. Local normal form of holomorphic functions, branch points.
- Conformal maps: isolated singularities, maps of the plane and of the punctured plane. Moebius transformations. Automorphisms of the extended plane.
- Maximum modulus theorem. Schwarz's lemma. Automorphisms of the unit disk.
- Riemann mapping theorem.
- Montel's and Vitali's theorems. Holomorphic functions defined by integrals: proper and improper case.
- The Gamma and Beta functions. Wielandt's uniqueness theorem. Euler's supplement. Multiplication formula.
- Infinite products, infinite products of holomorphic functions. Weierstrass' Delta function and Gauss' product formula.
- Approximation by rational functions. Bounded and unbounded connected components. Pole shifting lemma. Exhaustion of compacts subsets. Runge's theorem for compact subsets and for open subsets. Runge's little theorem.
- Characterization of simply connected domains.
- Principal parts distributions, Mittag-Leffler's theorem, partial fractions decompositions. The sine and the cotangent series. Summation of series by residues.
- Divisors and meromorphic functions on the plane and on the extended plane. The Picard group. Weierstrass' product and factorization theorems.
- Ideal theory for the ring of holomorphic functions: great common divisors, Wedderburn's lemma, finitely generated and prime ideals, closed ideals.

- Riemann's Zeta function. Bernoulli's numbers and Euler's identities. Euler's product formula. Integral representation of the Zeta function. Riemann's relation. Zeros of the Zeta and the critical strip.
- The Prime Number Theorem. Chebyschev's function. The Laplace and Mellin transforms.
