

COMPLEX ANALYSIS

Academic year 2020-2021

Detailed syllabus

Suggested references:

- teacher's notes (available on the webpage);
 - J.-P. Schneiders, *Fonctions de Variables Complexes*. Université de Liège (2010);
 - G. De Marco, *Selected Topics of Complex Analysis*. Università di Padova (2012);
 - T. W. Gamelin, *Complex Analysis*. UTM, Springer-Verlag (2001);
 - W. Rudin, *Real and Complex Analysis, Third edition*. McGraw-Hill (1986);
 - R. B. Ash, W. P. Novinger, Chapter 7 of *Complex Variables. Second Edition*. Dover (2007).
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Whenever not specified, all arguments are intended with proofs.

- Cauchy's argument principle. Rouché's and Hurwitz's theorems.
 - Open mapping theorem. Local injectivity and (local) biholomorphisms. Local normal form of holomorphic functions, branch points.
 - Conformal maps: isolated singularities, maps of the plane and of the punctured plane. Moebius transformations. Automorphisms of the extended plane.
 - Maximum modulus theorem. Schwarz's lemma. Automorphisms of the unit disk.
 - Riemann mapping theorem.
 - Montel's and Vitali's theorems. Holomorphic functions defined by integrals: proper and improper case.
 - The Gamma and Beta functions. Wielandt's uniqueness theorem. Euler's supplement. Multiplication formula.
 - Infinite products, infinite products of holomorphic functions. Weierstrass' Delta function and Gauss' product formula.
 - Approximation by rational functions. Bounded and unbounded connected components. Pole shifting lemma. Exhaustion of compact subsets. Runge's theorem for compact subsets and for open subsets. Runge's little theorem.
 - Characterization of simply connected domains.
 - Principal parts distributions, Mittag-Leffler's theorem, partial fractions decompositions. The sine and the cotangent series. Summation of series by residues.
 - Divisors and meromorphic functions on the plane and on the extended plane. The Picard group. Weierstrass' product and factorization theorems.
 - Ideal theory for the ring of holomorphic functions: great common divisors, Wedderburn's lemma, finitely generated and prime ideals, closed ideals.
 - Riemann's Zeta function. Bernoulli's numbers and Euler's identities. Euler's product formula. Integral representation of the Zeta function. Riemann's relation. Zeros of the Zeta and the critical strip.
 - The Prime Number Theorem. Chebyshev's function. The Laplace and Mellin transforms.
 - Schwarz's integral formula; Schwarz's reflection principle (weak and strong form).
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