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On the first positive eigenvalue of fourth-order Steklov and Neumann problems

Abstract

We consider the Steklov and Neumann eigenvalue problems for the biharmonic operator introduced respectively in [2] and [3], defined on a bounded domain Ω in $\mathbb{R}^N$ of class C^1.
For $N = 2$ these problems model a free vibrating thin plate under lateral tension, whose displacement at rest is described by the domain Ω and whose mass is displaced respectively at the boundary $\partial\Omega$ (Steklov problem), and on the whole of Ω (Neumann problem). The eigenfunctions represent the natural modes of vibration of the plate while the eigenvalues are the corresponding tones. We prove that among all bounded domain of class C^1 with a fixed volume, the ball maximizes the fundamental tone (the first positive eigenvalue) of the biharmonic Steklov problem (the maximization of the fundamental tone of the biharmonic Neumann problem has been proved in [3]).
Moreover, we prove convergence of the spectrum of a class of Neumann-type problems for the biharmonic operator to the spectrum of the Steklov problem under consideration, highlighting the strict connection among these two problems.
Finally, we prove a quantitative version of the isoperimetric inequalities for the fundamnetal tone of both Steklov and Neumann problems, showing that such inequality is sharp.

This talk is based on the results of the papers [1, 2, 4].

Bibliography

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