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Shape optimization for the eigenvalues of a Biharmonic Steklov problem.

Abstract

Let Ω be a bounded domain in $\mathbb{R}^N$ of class C^1 and $\tau > 0$ be fixed. We consider the following Steklov-type eigenvalue problem for the biharmonic operator

$$
\begin{cases}
\Delta^2 u - \tau\Delta u = 0, & \text{in } \Omega, \\
\frac{\partial^2 u}{\partial\nu^2} = 0, & \text{on } \partial\Omega, \\
\tau\frac{\partial u}{\partial\nu} - \mathrm{div}_{\partial\Omega}\big(D^2 u.\nu\big) - \frac{\partial(\Delta u)}{\partial\nu} = \lambda u, & \text{on } \partial\Omega,
\end{cases}
\tag{1}
$$

in the unknowns u (the eigenfunction) and λ (the eigenvalue). For $N = 2$ this problem models a free vibrating thin plate under lateral tension τ whose displacement at rest is described by the domain Ω and whose mass is concentrated at the boundary $\partial\Omega$. The eigenfunctions represent the natural modes of vibration of the plate while the eigenvalues are the corresponding tones. We prove that balls are critical domains for the symmetric functions of the eigenvalues under isovolumetric perturbations of the domain Ω. Moreover, we prove that among all bounded domain of class C^1 with a fixed volume, the ball maximizes the first positive eigenvalue. Finally, we highlight the strict relation between problem (1) and a Neumann-type eigenvalue problem for the Biharmonic operator by showing convergence of the spectrum of a family of Neumann-type problems for the Biharmonic operator to the spectrum of problem (1).

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