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On the eigenvalues of a fourth-order Steklov problem

Abstract

Let Ω be a bounded domain in $\mathbb{R}^N$ of class C^1 and $\tau > 0$ be fixed. We consider the following Steklov-type eigenvalue problem for the biharmonic operator

$$
(1) \qquad \begin{cases} \Delta^2 u - \tau\Delta u = 0, & \text{in } \Omega, \\ \frac{\partial^2 u}{\partial \nu^2} = 0, & \text{on } \partial\Omega, \\ \tau\frac{\partial u}{\partial \nu} - \mathrm{div}_{\partial\Omega}\big(D^2 u.\nu\big) - \frac{\partial(\Delta u)}{\partial \nu} = \lambda u, & \text{on } \partial\Omega, \end{cases}
$$

in the unknowns u (the eigenfunction) and λ (the eigenvalue). For $N = 2$ this problem models a free vibrating thin plate under lateral tension (the parameter τ) whose displacement at rest is described by the domain Ω and whose mass is concentrated at the boundary $\partial\Omega$. The eigenfunctions represent the natural modes of vibration of the plate while the eigenvalues are the corresponding tones. We prove that among all bounded domain of class C^1 with a fixed volume, the ball maximizes the fundamental tone (the first positive eigenvalue). Moreover, we prove convergence of the spectrum of a class of Neumann-type problems for the biharmonic operator to the spectrum of problem (1), highlighting the strict connection among these two problems.

Bibliography

[1] C. Bandle, *Isoperimetric inequalities and applications.* Pitman advanced publishing program, monographs and studies in mathematics, vol. 7, 1980.

[2] L.M. Chasman, *An isoperimetric inequality for fundamental tones of free plates.* Comm. Math. Phys. 303 (2011), no. 2, 421449.

[3] D. Buoso, L. Provenzano, *An isoperimetric inequality for the first non-zero eigenvalue of the biharmonic Steklov problem*, in preparation.

[4] P.D. Lamberti, L. Provenzano, *Viewing the Steklov eigenvalues of the Laplace operator as critical Neumann eigenvalues*, submitted to the 9th Isaac Congress proceedings.