## ***Neumann vs Steklov: an asymptotic analysis for the eigenvalues.***

We consider the spectral problem

$$\begin{cases} -\Delta u_\varepsilon = \lambda_\varepsilon \rho_\varepsilon u_\varepsilon, & \text{in } \Omega, \\ \frac{\partial u_\varepsilon}{\partial \nu} = 0, & \text{on } \partial\Omega \end{cases}$$

in a smooth domain $\Omega$ of $\mathbb{R}^2$. The first equation involves a parameter $\rho_\varepsilon$ which is a positive measurable function and plays a role of a mass density. We assume that $\rho_\varepsilon$ is piecewise constant and is of order $\varepsilon^{-1}$ in a $\varepsilon$-neighborhood of the boundary, as $\varepsilon$ goes to zero, and is of order $\varepsilon$ in the rest of $\Omega$. Moreover its integral over the whole of $\Omega$ is fixed and does not depend on $\varepsilon$. We study the asymptotic behavior of the eigenvalues and the eigenfunctions as $\varepsilon$ goes to zero and obtain explicit formulas for the first and second order terms of the corresponding asymptotic expansions in terms of solutions of suitable auxiliary boundary value problems. In particular, it turns out that the Neumann eigenvalues converge to the appropriate Steklov eigenvalues as $\varepsilon$ goes to zero. Moreover, we find a closed formula for the so-called *topological derivative* of the eigenvalues at $\varepsilon = 0$. Finally, in the case that $\Omega$ is the unit ball, we prove that the derivative of all the eigenvalues at $\varepsilon = 0$ is positive and then we conclude that *"the Steklov eigenvalues locally minimize the Neumann eigenvalues"* for $\varepsilon$ small enough.
All the results contained here can be found in [1, 2, 3, 4].

### Bibliography


[1] D. Buoso, L. Provenzano, A few shape optimization results for a biharmonic Steklov problem, Journal of Differential Equations (259), 5 (2015), 1778–1818.

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[4] P.D. Lamberti, L. Provenzano, Neumann to Steklov eigenvalues: asymptotic and monotonicity results, submitted.