Mean Field Games: a short introduction

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Abstract. I will present the motivations and the basic equations of the recent theory of Mean Field Games, following Lasry and Lions [1]. The goal is describing non-cooperative stochastic differential games with a continuum of identical players behaving according to rational expectations. The approach is based on letting the number N of players go to infinity in Nash feedback equilibria of N-person games where the coupling of the players is only in the cost functional. A new system of PDEs is proposed for these games: they are N Hamilton-Jacobi-Bellman equations of second order for the value of each player coupled with N Kolmogorov-Fokker-Plank equations for the equilibrium distributions. Letting $N \to \infty$ one obtains a system of only two equations, a HJB and a KFP, which are called MFG equations.

Explicit solutions of such systems of PDEs can be obtained if the dynamics of each player is linear and the cost criterion is quadratic in the state and the control [2]. This allows a simple discussion of the dependence of solutions on the parameters, in particular for some reference models of population distribution. It also shows the connection between the MFG theory of Lasry and Lions and the "Nash certainty equivalence principle" for large-population cost-coupled LQG problems introduced and developed by Caines, Huang, and Malhamé for wireless power control and other engineering problems.

[1] J.-M. Lasry, P.-L. Lions: Mean field games. Jpn. J. Math. 2 (2007), 229–260.

[2] M. Bardi: Explicit solutions of some Linear-Quadratic Mean Field Games, preprint 2011, submitted.

[3] M. Huang, P.E. Caines, R.P. Malhamé: Large-population cost-coupled LQG problems with nonuniform agents: individual-mass behavior and decentralized ϵ-Nash equilibria. IEEE Trans. Automat. Control 52 (2007), 1560–1571.