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An Italian perspective on the development of financial mathematics from 1992 to 2008

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Abstract
This paper is intended to be a survey of the development of financial mathematics as seen through the events that I organised, and partly co-organised, between 1992 and 2008. These events all took place in Italy between 1992 and 2003, while in 2008 I was involved in the organisation of an entire special semester in Linz (Austria); this semester is included here because it marks quite well the state-of-the-art of the period just before the so-called big financial crisis that lasted from, roughly, 2008 to 2012. Even if the survey may be affected by my personal views, it can still be seen as reflecting the actual global development since what I am going to describe here concerns major occurrences. For completeness, I also mention, although only briefly, some events that took place in Italy during the given period, but where I was not personally involved.

Keywords Financial mathematics · Basic scientific aspects · Historical and organisational aspects · Scientific institutions · Big financial crisis

Mathematics Subject Classification (2010) 91-03 · 01A99 · 91G99

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1 Introduction

The development of a scientific field can best be seen from the output of scientific work, and this depends on the availability of scientific journals to disseminate it properly. It also depends on the possibility of having scientific meetings where people may exchange their views and where they can make their work known to others. This in particular allows one to gauge the directions into which a field is moving and to understand "what people think about what people think". For scientific areas connected with applications, the directions into which to move are best understood by actual

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contacts with practitioners or through internships, but also meetings can make one aware of them. The developments depend furthermore on the availability of specific schools/courses that allow younger people to access the field more easily. For the organisation of meetings and schools, an important role is played by scientific institutions or societies that provide support and funding.

Having been involved in the organisation of a series of international conferences and schools in the area of financial mathematics between 1992 and 2008, I should like to describe here how the field evolved during this period as seen through these conferences and schools. I am focusing here mainly on activities that took place in Italy between 1992 and 2003 and in which I was personally involved, adding some information on a special semester that took place in Austria in 2008 and of which I had the honour to chair the scientific committee. In this way, I am also able to span an interesting and intense period: Around 1992, most mathematically oriented studies concerning finance were based on the seminal works of Harrison, Kreps and Pliska, see e.g. [11, 12] (see also the textbook by Duffie [9] for the state-of-the-art at that time), but the further seminal works by Delbaen and Schachermayer (e.g. [7, 19, 8]) were already in the coming. It was then that in the context of finance, a new more academic and mathematically oriented discipline started to develop under the names of *financial mathematics* or *mathematical finance*. It was also around that time that the important Bachelier Finance Society was created and the journal *Finance and Stochastics* was launched. On the other hand, the year 2008 was when the so-called big financial crisis started, which lasted for about four years, and during which some major changes took place in the financial industry. It is also in this latter sense that 2008 is a good moment where to stop this survey.

Inevitably, the survey that I am going to present may be influenced by my personal views, and I apologise from the beginning for possible resulting shortcomings. Furthermore, in various cases, I had to make a selection of what to present and here too I might have overlooked some important points.

To briefly introduce my own connection with financial mathematics and to explain my organisational involvement, I should like to mention that, like other colleagues in financial mathematics of my generation, I had originally been working in stochastic filtering and control. In stochastic filtering problems, one has essentially to compute the conditional expectation of (a function of) a signal process, given the filtration generated by noisy observations of the signal. Analytic solutions are available for the so-called linear filtering problems; for nonlinear problems, one must typically resort to approximations. One convenient tool for filter approximations is a measure transformation from the original physical measure to a fictitious measure, under which the observations become independent of the signal while the marginal distribution of the signal remains the same as under the physical measure. In this way, the conditional expectation reduces to an ordinary expectation with the observations entering simply as an input, thus making it easier to perform the approximations. In 1984, I was visiting Alain Bensoussan at INRIA and one day, he gave a seminar on his paper [2]. In my opinion, in this paper, Bensoussan succeeded in clarifying, in the context of a continuous-time market model, how many hedging instruments one needs with respect to the independent exogenous sources of randomness so that the market is complete. The basic idea was already present in Harrison and Pliska [13], but

[2] makes the economic content more apparent in analogy to what was known from the discrete-time and finite-sample-space case (see Cox, Ross and Rubinstein [6]). As a tool, Bensoussan used measure transformation that I was familiar with, and so I started getting interested in financial mathematics. With my former student Paolo Kind, who at that time was at the Department of Economics in Chicago, and with the technical support of Robert Liptser, we wrote the paper [14] concerning limiting properties of a path-dependent local volatility model that appeared in the newly established journal *Annals of Applied Probability*, for which Yannis Karatzas was serving as Associate Editor and was soliciting contributions in financial mathematics. This paper earned me an invitation to organise a first conference in Erice (see Sect. 3.1 below), and from then on up to 2003, I (co-)organised conferences on financial mathematics in Italy almost yearly, with major conferences taking place roughly every two years, in Erice, Cortona and Trento. In-between, there were two major schools in Brixen/Bressanone (the double name of the town comes from the fact that it lies in a geographical area that was originally Austrian and after World War I became part of Italy). Details on these conferences and schools are given below.

As already mentioned, in order to cover the period up to the big crisis, I should like to mention in this survey also the activities that took place in 2008 in Linz (Austria) and where I was involved as chairman of the scientific committee. The reason is that in my view, this semester was one of the major events in financial mathematics during the mentioned period. I also have to point out that what I am going to describe here concerning the activities in financial mathematics in Italy is related to my own organisational activity, except for what I shall briefly mention in Sect. 5.

The rest of this survey is organised as follows. Since conferences and schools do not just come "out of the blue", but need funding and organisational support, I should like to give in Sect. 2 some information regarding the Italian scientific organisations that were crucial for the actual implementation of the conferences and schools that I (co-)organised. The information on the supporting organisation for the special semester in Linz in 2008 is given directly in the description of the semester itself in Sect. 3.4. I should like to take here the opportunity to express my gratitude towards these institutions, more precisely towards the people who promoted them and were responsible for their activities; they always had an open eye also towards our field of financial mathematics. In this same Sect. 2, I should finally like to take the opportunity to mention also less known facts concerning some connections with Italy of the scientific society/network AMaMeF that actually started in Italy in 2005, as well as of the journal *Finance and Stochastics* itself, for which some preliminary initiatives took place in Italy before its actual launching in 1996. In Sect. 3, I shall then enter into more detail for what concerns the conferences that I (co-)organised. Their description is arranged according to the timing of the conferences and grouped according to their location/venue. For each location, a brief description is given of the venue, occasionally accompanied by some anecdotes. This is then followed by a description of the scientific part. Since the activities took place in a relatively distant past, I could not always recover sufficient material to work out a full scientific review. In those cases, I shall at least mention the topics/titles of the various presentations as they still give some indication on what was going on. The description for each of the conferences is followed by a list of names that, to keep it at a reasonable length,

is limited to the names of the main invited lecturers. In Sect. 4, I shall then come to describe the schools that I am dividing into two groups with the groups differing for their venue as well as for their structure. The first group of schools took place in Cortona, a venue also of two of the conferences and therefore described there. For the second group, that took place in Brixen/Bressanone, a brief description of the venue is also provided. Section 5 describes briefly some other activities in financial mathematics that took place or were started in Italy during the period considered here. Section 6 contains some concluding remarks that on the one hand give more details on how everything came about and on the other hand can be seen as a summary of the more detailed descriptions in Sects. 3 and 4. The bibliography at the end is not intended to be exhaustive to this survey, but as it stands, it is included for convenience of presentation.

2 Supporting scientific institutions and further comments

2.1 Italian scientific institutions that supported conferences in financial mathematics

Major funding for the Cortona conferences came from GNAFA (Gruppo Nazionale Analisi Funzionale ed Applicazioni), a subcommittee of what was then the mathematics committee of the CNR (Consiglio Nazionale delle Ricerche). Together with INdAM (Istituto Nazionale di Alta Matematica), it was the reference organisation for the funding of activities not only in mathematical analysis and applications, but also in probability and its applications. After a reorganisation, it has now slightly changed the name and is a subgroup of INdAM.

The conference in Trento was organised within the activities of the CIRM (Centro Internazionale per la Ricerca Matematica) of the Istituto Trentino di Cultura (now Fondazione Bruno Kessler). Note that the acronym CIRM here should not be confused with the same acronym referring to the well-known conference centre in Luminy (Marseille). The Istituto Trentino di Cultura is being supported by funds from the local governement, the Provincia Autonoma di Trento. It is a local scientific organisation founded in 1978 and intended to be an institution, auxiliary to the University of Trento, capable of attracting highly qualified researchers in the general area of mathematics and its applications. The main activity of CIRM is the organisation of seminars and workshops lasting generally for one week. Up to now, 326 events have been organised with the participation of some 18'000 mathematicians from all over the world. CIRM does not have a specific venue for its conferences, most of them being held in a hotel in a nearby township.

Although it did not directly support the activities that I (co-)organised myself, a major Italian institution that promotes research in areas of mathematics as applied also to finance is AMASES (Italian Association for Mathematics Applied to Social and Economic Sciences). It was founded in 1976 and together with the two further institutions SIMAI and AIRO forms the Italian Federation of Applied Mathematics (FIMA). I am mentioning it here as well because AMASES was running its own journal since 1978 and this journal was internationalised in 2000 to become the present-day *Decisions in Economics and Finance: A Journal of Applied Mathematics*.

2.2 Italian scientific institutions that supported schools in financial mathematics

The schools in Cortona took place under the sponsorship and the funding of SMI (Scuola Matematica Interuniversitaria), the scope of which is to provide young researchers with basic training in mathematical techniques for applications. This is implemented through summer schools in Perugia (general basic training) and Cortona (schools on more advanced specific topics). SMI started its activities in 1971, and on three occasions, one of the courses in Cortona was intended for mathematical techniques in finance, more specifically in 1997, 2006 and 2011. The sponsoring institution for the two schools in Brixen/Bressanone was the CIME foundation (Centro Internazionale Matematico Estivo), one of the early (1954) scientific institutions in Italy to organise schools in the general area of mathematics. Typically, the CIME schools are summer schools consisting of about 4–5 invited lecture series by established experts in addition to a certain number of seminars, mostly by the participants themselves. The lectures are published in the CIME Foundation sub-series of the *Springer Lecture Notes in Mathematics*. Also for the two schools on financial mathematics in Brixen/Bressanone, the lectures are published in this series as Volume 1656 and 1856, respectively (see [18, 10]).

2.3 Connections with Italy of one specific scientific institution in financial mathematics

The Bachelier Finance Society (BFS) may well be considered as the main scientific society in the field of financial mathematics. Especially for European scientists, there is, however, also another scientific society, actually a network for researchers active in the field, named AMaMeF (Advanced Mathematical Methods for Finance). It started in 2005 as a project with the European Science Foundation upon an initiative by Roberto Natalini, the present director of the laboratory IAC of the CNR, who was project coordinator for the first three years, while Bernt Øksendal became the scientific director. Initially, the website of AMaMeF was based at IAC; later, it was moved to Oslo and Giulia Di Nunno became the reference person; presently, it is coordinated by Robert Stelzer. The network is spanned across Europe with members affiliated with more than 20 countries (see http://amamef.impan.pl).

2.4 A comment relating to the journal *Finance and Stochastics*

By 1995, the period when the BFS was founded, the field had already strongly evolved also among European researchers, and so the idea came up to start a more European-based journal next to the already existing US-based journal *Mathematical Finance*. This idea was pursued by some leading experts, among them Yuri Kabanov, who was planning to have such a journal based in Padova. I was somehow reluctant, insisting that we needed to be backed by an established publisher. It was then Dieter Sondermann who was able to involve the Springer-Verlag, and so the new journal *Finance and Stochastics* was launched in 1996 with Springer as publisher and Sondermann as first Editor-in-Chief.

3 The conferences

In this section, I am going to describe the conferences that, mostly together with a co-director, I organised in Italy between 1992 and 2003, adding in Sect. 3.4 the description of the special semester in Linz in 2008, where I was involved as chairman of the scientific committee.

3.1 Erice 1992

3.1.1 The place and some anecdotes

Erice is a rather special place situated on the top of a mountain at the westernmost tip of Sicily with the top occasionally lost in the clouds. Looking up from the bottom, this could then remind one of the Olympus mountain on the top of which lived the gods. During the conference, the gods were of course us. The venue there is called "Centre for Scientific Culture" and consists of various buildings, some of them being former monasteries. The centre is run by physicists and named after Ettore Majorana, a physicist from E. Fermi's group, who disappeared in 1938 after a boat trip from Naples to Palermo. The organisers at the Centre try their best to avoid any kind of problems for the participants especially in difficult times. In particular, for the conference in 1992, the lecturers were picked up individually by taxi at the airport and also individually driven back. It happened that the highway on which Yannis Karatzas was driven to the airport in the morning of May 23 was blown up about two hours later in an attack aimed to kill the judge Giovanni Falcone; but Yannis had been driven safely. Also the excursion was organised so that the safety of the participants could be guaranteed; we had a memorable excursion to one of the famous archeological sites of Sicily, the Valley of the Temples. The centre was highly valued among the population of Erice and it was easier to get cash in the bank with a private check countersigned by the centre, rather than by card. Finally, a very special feature of the conferences there, at least at that time, was that participants could freely choose among the various restaurants in town where to have their meals. Food was free of charge, only drinks had to be paid for. In exchange, in the evenings, there was a small barrel of Vin Santo wine in one of the common rooms and participants could freely pour wine from the barrel; occasionally, it was completely emptied. To stimulate restaurants to provide a good service, the organisers of the various conferences were in charge of identifying the best one; this would then be made the venue for the social dinner.

3.1.2 The conference in general

The conference took place from May 14 to 22, 1992 and its purpose was to assess the state-of-the-art, at that time, of the contribution that mathematical methodology, in particular stochastic and optimisation methods, was giving to the solution of problems arising in mathematical economics, with major emphasis on financial mathematics. At the time of the conference, the fundamental results of Delbaen and Schachermayer were not yet generally known, and so the setting was in a sense pre-Delbaen–Schachermayer, based mainly on the theories developed by Harrison, Kreps

and Pliska. Participants came prevalently from academia, but there were also some practitioners present coming from the World Bank, the Bank of America, Banque de France, Banca d'Italia as well as from regional banks and corporate research units.

Due to organisational reasons, in parallel to the more specifically mathematical finance conference, there were also presentations by members of a delegation of five scientists from the Division of Mathematical Economics of the Institute of Systems Science from the Academia Sinica in Beijing. These presentations were more specifically related to topics from economics, among which equilibrium of economies of small-scale efficiency.

The abstracts of the talks of the mathematical finance part of the conference and the full papers by the Chinese delegation are contained in the booklet [17].

3.1.3 The conference, scientific aspects

The presentations during the conference concerned the topics as listed below, which represent some of the hot topics at the time. For each topic, I am going into a bit more detail for what concerns the actual contents of the individual presentations:

- *Price formation:* microeconomic point of view with distinction between informed traders and noise traders; habit formation in a general-equilibrium representative-agent stochastic production economy.
- *Price evolution models:* relationships between discrete-time and continuous-time models; statistical analysis of stock exchange data to support Lévy versus Wiener modelling; hidden Markov models.
- *Option valuation:* pricing in models with Markov volatilities; pricing in incomplete markets and the minimal martingale measure; forward-starting Asian options.
- *Hedging:* hedging via results on constrained portfolio optimisation; mean–variance hedging approaches.
- *Portfolio optimisation and multi-period decision making:* approach via convex duality, also for problems with constraints; portfolio optimisation with fixed transaction costs; investment and consumption with labour income.
- *Interest rates and control of the exchange rate:* parametric and non-parametric estimation in term structure models; various approaches to the control of the exchange rate.
- *Numerical methods:* applications of the stochastic Taylor formula with higher order weak and strong schemes to the pricing in incomplete markets with stochastic volatility.
- *Practitioner contributions:* among others, valuation of corporate credit risk at Bank of America; analysis of the Italian treasury and bond market.

The main lecturers for the mathematical part were (in alphabetical order) R. Avesani (33), E. Briys, R. Cesari (13), J. Detemple (7), J. Dupačová (18), E. Eberlein (19), R. Elliott, H. Föllmer (8), M. Jeanblanc-Picqué (27), Yu. Kabanov (23), Y. Karatzas (15), P. Lakner, F. Moriconi (14), E. Platen (9), S. Pliska (22), M. Schweizer (17), D. Talay (21), A. Venetoulias. The numbers in brackets identify them in the group picture in Fig. 1.

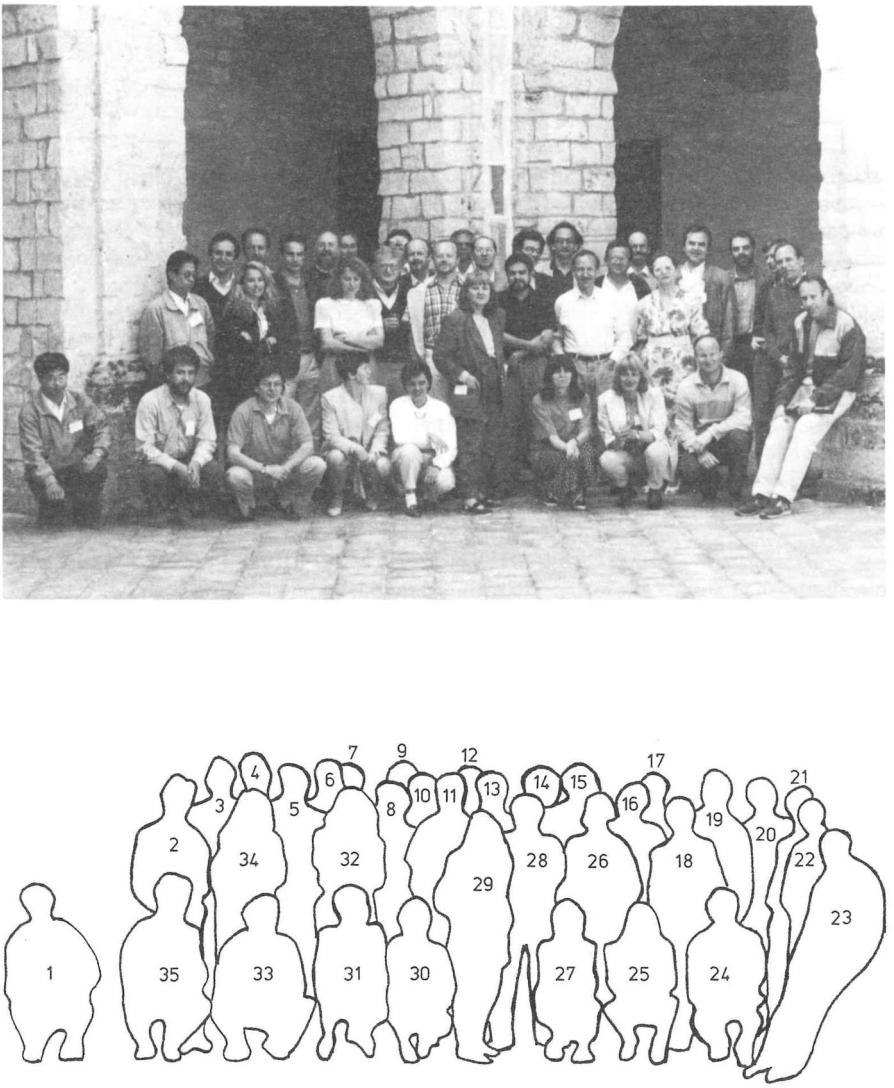

Fig. 1 Erice

3.2 Cortona 1994 and 1998

3.2.1 The place and some anecdotes

The two conferences as well as the schools in Cortona took place in what is called the "Palazzone". It is an imposing medieval building, restored into a Renaissance Palazzo, that had various owners over the years and was ultimately donated to the Scuola Normale of Pisa in 1968. The Scuola uses it mainly for conferences and schools. It is situated a bit outside of the town of Cortona, which is a very beauti-

ful little township in Tuscany, overlooking the Val di Chiana, and has a very pleasant climate. Participants are accommodated in the Palazzone itself up to filling capacity, others in a nearby residence or in a hotel in town. The Palazzone also has a tower, and up to a certain date, more daring participants could be accommodated there as well. This was still possible for the 1994 conference and Chris Rogers opted for the topmost room in the tower, thus earning the "title" of Prince Chris; on the very top, there was also space for leisurely gatherings. There are unfortunately no restaurants or bars in the close vicinity. Lunch was provided by a caterer, but for dinner, one had to go to town. On the one hand, this turned out to be a bit problematic in the beginning because then the Palazzone would close at 10.30 pm whereas dinner in town would not be served before 8–9 pm; later on, residents of the Palazzone were given a key to enter it in the evenings. On the other hand, this forced participants to take at least one walk a day, and the evenings in town were very suggestive.

Reaching the Palazzone is also not fully straightforward. Cortona can essentially be reached by train (once there was the risk of a train strike on the first day of one of the conferences) only by getting off at one of the stations down in the Val di Chiana. Participants were advised to take a taxi from there, but the information did not always go through properly. It thus happened that driving with my car down the road from Cortona to the Palazzone, I noticed Hans Föllmer walking the two kilometres with heavy luggage under the burning afternoon sun; he had been wrongly advised by a participant, whom he met at the train station, to take the bus to the centre of Cortona and then to walk from there. The luggage was heavy because he came directly from a meeting in Oberwolfach and had much paper material with him. As a further little anecdote concerning "Cortona", there is the story of a participant from Barcelona who was supposed to stay at the hotel S. Michele in town. In spite of his great interest, he showed up only on the second day: he had wrongly travelled to Cortina, a mountain resort in northeastern Italy. As he could not find a hotel S. Michele there, he realised that he went to the wrong place; it took him a full day to travel from Cortina to Cortona. I may add that there is also a town in Italy called Ortona.

Finally, it also happened that the period of the 1998 conference was overlapping with the period of the soccer world championship which then took place in France. Was it more interesting to the participants to attend lectures or watch soccer? The problem could be solved since the games were shown on TV from the later afternoon onwards. The schedule was slightly compressed and a TV set with a big screen was placed in one of the lecture rooms.

3.2.2 The conference in 1994, scientific aspects

The first of the two Cortona conferences took place in 1994 from May 29 to June 3 under the title "Recent Developments in Mathematical Finance" and was co-directed with Robert Elliott. The "hot topics" for this conference were perhaps not much different from those of the Erice conference two years earlier. The context had changed though, due mainly to the fact that by then, the Delbaen–Schachermayer approach had taken a solid hold. Most of the talks reflected front-line research as can be seen from the more detailed list below. The fact that less weight was given this time to portfolio optimisation is only coincidental, not because it would not anymore have

been a topic of major interest. On the other hand, the relationship between finance and insurance was attracting more and more attention. While the second Cortona conference in 1998 was putting a major emphasis on such connections, there were some contributions also in the 1994 conference. All in all, the conference reflected the state-of-the-art of financial mathematics in general at that time, focusing on the following topics:

- Concerning the *basic concepts* of arbitrage and martingale measures, there were two presentations under the titles "Asymptotic arbitrage in a model for a large financial market" and "On existence of martingale measures in finite security markets".
- At that time, issues concerning *price formation* and *price evolution models* were of major interest, and in this context, there were presentations under the titles "A microeconomic approach (informed and noise traders)", "Stock price models with feedback effect" and "Empirical results on German stock returns".
- *Options and their pricing* are among the main topics in financial mathematics, and it was at this conference that Mark Davis presented for the first time his utility-indifference pricing approach. There were various presentations under this heading with the following titles, where the first one can also be seen as a first version of the optional decomposition theorem: "Dynamic programming and pricing of contingent claims in an incomplete market", "Option pricing with a quadratic loss function", "Utility indifference pricing", "Nonstandard analysis methods in option pricing", "American options on interest rates" and "Numerical methods for the computation of American option prices".
- *Interest rates* are quite central to financial mathematics and here too, there were various presentations, among them also from people involved in their applications within the Italian market and the government. Presentations were made under the titles "Hedging of interest rate risk", "Fitting an interest rate model to data", "A model with singular interest rates", "A multinomial interest rate model for the Italian market" and "Relationship between liquidity and short term interest rates in the Italian inter-banking market".
- While the relationship between *finance and insurance* was given a major emphasis during the second Cortona conference (see below), there were two presentations also here under the titles "An Asian option approach to the valuation of insurance futures contracts" and "Valuation of index-linked life insurance policies with minimum guarantees by stochastic models for real and nominal interest rates".
- *Portfolio optimisation* was not a major topic during this conference; there was, however, an interesting presentation under the title "Asymptotic turnpike results in portfolio optimization".
- *Applications* are always of interest. There were unfortunately just two presentations in this direction, which showed, however, that the *green economy* was already about to become a major issue. The titles were "The evaluation of pollution permits in a stochastic economy with quotas" and "Searching for fractal structure in agricultural futures markets".

The main lecturers were (in alphabetical order) G. Barone-Adesi, M. Davis, E. Eberlein, L. Élie, N. El Karoui, R. Elliott, H. Föllmer, H. Geman, E. Kopp, D. Lamberton, F. Moriconi, Yu. Kabanov, E. Platen, C. Rogers, M. Schweizer.

3.2.3 The conference in 1998, scientific aspects

The second Cortona conference took place in 1998 from June 8 to 12 under the title "The Interplay between Finance, Insurance and Statistics" and was co-directed by Ole Barndorff-Nielsen and Mauro Pagliacci.

Over the years, the areas of finance and insurance were increasingly sharing problems, concepts and methods, in particular for what concerns the control of risk. Also the application of statistical methods had much in common in the two areas. It was thus felt that the moment had come to have a conference connecting these areas. The conference concentrated on the following topics:

- Various talks were related to *insurance and its relations to finance*. The titles were "Finance and actuarial science: two solitudes no longer", "Financial risk in insurance and viceversa", "Alternative risk transfer: some thoughts, some examples", "Hedging and reserving for segregated fund guarantees", "Optimal strategies for insurers: investment, reinsurance, and new business", "Ruin problem from the viewpoint of large deviations theory" and "Modeling catastrophe insurance products based on actually reported claims".
- *Modelling and statistical analysis* of structured stochastic models is important in both finance and insurance, as is the empirical verification of specific models. In this context, the talks carried the following titles: "Aggregation and model construction for volatility models", "On the interplay between cross-sectional and intertemporal parameters in finance", "A global market model" and "More realistic modeling in finance".
- *Interest rates* are also important in both areas. At that time, one hot topic was the finite-dimensional representation of interest rate dynamics that was addressed in the first of two talks on interest rates, "A geometric approach to interest rate theory" and "Eurofutures: towards a multivariate factor model of a leading interest rate market".
- *Valuation and hedging* belong to the basic problems both in finance and insurance. The three talks here concerned "Valuation of average options", "On credit risk valuation" and "Hedging strategies with bounded risk".

The main lecturers here were (always in alphabetical order) T. Björk, Ph. Boyle, B. J. Christensen, D. Duffie, D. Dufresne, E. Eberlein, P. Embrechts, H. Föllmer, M. Hardy, C. Hipp, U. Müller, R. Norberg, H. Nyrhinen, F. Ortu, E. Platen, H. Schmidli, N. Shephard.

3.3 Trento

3.3.1 The place/venue

This conference that took place from May 26 to 30 in 1997 was called "3rd Italian Conference on Mathematical Finance" because it was preceded by that in Erice and that in Cortona in 1994 as major international conferences in Italy in the area of financial mathematics. It was supported by CIRM (see Sect. 2.1) and co-directed by Luciano Tubaro from the University of Trento as well as by Renzo G. Avesani, at that

time from the bank Cariplo in Milan. Most of the workshops/conferences supported by CIRM take place in a hotel in a nearby town. This has the advantage of keeping the participants together also outside of the official schedule. For this specific conference in financial mathematics, I had opted to accommodate the participants in hotels in the city of Trento itself, while the venue was in a newly established campus of IRST-ITC, a local cultural institution with the same parent organisation as CIRM. This campus is located in Povo on a beautiful hillside overlooking the city of Trento and the Adige valley with some Dolomites-type mountains in the background.

3.3.2 The conference, scientific aspects

The conference encompassed again a rather wide range of topics that appeared to be of major interest at that time. The themes of the presentations can be arranged according to the following topics:

- *Basics*: Presentations under the titles "Arbitrage theory", "Incomplete markets", "Constructing a consistent market model" and "The generalised numéraire portfolio".
- *Options and contingent claim pricing*: Here the titles of the presentations were "Option pricing with regularised Brownian motions", "Certainty equivalent and martingale pricing in incomplete markets", "Pricing contingent claims with bid–ask spreads: an axiomatic approach", "Some remarks on option pricing with a volatility range", "Does convergence of stock price processes imply convergence of option prices?", "Approximation of look-back type options" and "The American option embedded in the Italian saving bonds".
- *Price evolution models*: The titles were "Some financial models with Poisson point processes" and "Enlargement of filtration and modelling of a discontinuous market from an insider point of view".
- *Interest rates*: The titles here were "Interest rate models and consistent forward curve families", "LIBOR and swap market models and measures", "Models of bond prices, LIBOR and swap rates" and "Some methodological aspects of modelling interest rate uncertainty".
- *Hedging, risk minimisation and asset management*: The titles were "Hedging and liquidation under transaction costs in currency markets", "Hot spots and hedges", "Local risk minimisation under transaction costs", "Locally risk minimising strategies for American options" and "Risk sensitive dynamic asset management".
- *Insurance and extreme events*: Titles here were "Diffusion models for dividend pay-out optimisation for a firm with random risk process" and "Modelling extremal events in finance".

The main invited speakers were G. Barone-Adesi, T. Björk, F. Delbaen, P. Embrechts, M. Frittelli, A. Grorud, F. Jamshidian, M. Jeanblanc-Picqué, E. Jouini, Yu. Kabanov, D. Lamberton, R. Litterman, C. Martini, F. Moriconi, M. Musiela, F. Ortu, E. Platen, S. Pliska, M. Pontier, M. Pratelli, W. Schachermayer, D. Sondermann, M. Taksar, D. Talay.

Most of these speakers may be spotted in the group picture in Fig. 2.

This conference was attended by 120 participants and can be connected to the series of CIRM conferences on "Stochastic Partial Differential Equations and Applications" that took place over various of the previous years and is still continuing.

Fig. 2 Trento

3.4 The Special Semester in Linz

3.4.1 The place/venue

The Semester took place at RICAM in Linz (Austria). RICAM (Johann Radon Institute for Computational and Applied Mathematics) was founded in 2003 as an institute of the Austrian Academy of Sciences and focuses on basic research in applied mathematics. It is located on the campus of the Johannes Kepler University in Linz. Among its various activities are the "Special Semesters" focusing on various topics of applied mathematics. Upon suggestion by Heinz Engl, the founder of RICAM, the "Special Semester on Stochastics with Emphasis on Finance" was organised there from September to December 2008.

3.4.2 The Semester in general

The scientific organisation was entrusted to a committee that included some RICAM affiliates and, in particular for finance and insurance, Hansjörg Albrecher and Walter Schachermayer in addition to myself as chair. The Semester was structured to start with an initial "Autumn School" on the various topics of the Semester intended mainly as a tutorial for the benefit of younger participants. It was followed by a "Kick-off Workshop" as well as a series of thematic workshops and closed by a "Concluding Workshop". During the same period, from September 17 to 19, there was a Summer School there on copulas, organised by the University. During the Semester, there were 124 presentations and a total of 222 participants.

3.4.3 The thematic workshops

For the four thematic workshops, I summarise below the topics that were touched upon by the presenters. Each of the workshops lasted for a full week:

– "Workshop on Advanced Modelling in Finance and Insurance": The lectures concerned a great variety of topics, among which modelling with Lévy processes, Brownian semi-stationary processes, market microstructure, stochastic volatility, asymptotic arbitrage, markets with transaction costs, optimal liquidation, risk measures and optimal investment, insurance products, energy markets.

  The lecturers for this workshop were O. Barndorff-Nielsen, F. E. Benth, E. Eberlein, H. Föllmer, D. Hobson, S. Jacka, J. Kallsen, C. Klüppelberg, S. Malamud, G. Pflug, M. Rásonyi, C. Rogers, A. Schied, H. Schmidli, R. Stelzer, J. Woerner.

– "Workshop on Optimisation and Optimal Control": A large portion of the talks concerned optimisation problems in an insurance context. Among other topics, there were stochastic target problems, investment under drawdown constraints, investment with habit formation, optimisation in market models with jumps, control of forward–backward stochastic differential equations, backward SDEs with constrained jumps, and quasi-variational inequalities.

  The lecturers for this workshop were N. Bäuerle, B. Bouchard, R. Elie, H.-U. Gerber, F. Gozzi, Ch. Hipp, D. Lamberton, B. Øksendal, J. Paulsen, H. Pham, M. Schäl, H. Schmidli, M. Steffensen, A. Sulem, P. Tankov, N. Touzi, M. Zervos, X. Y. Zhou, G. Žitković.

– "Workshop on Inverse and Partial Information Problems": Various issues in financial mathematics are related to inverse problems (e.g. calibration can be considered as an inverse problem), but they are generally not referred to under this name. There is a strong group on inverse problems at RICAM; the founder of RICAM, Heinz Engl, is himself an expert in the field, and so it was natural to have a thematic workshop referring explicitly to inverse problems. Such problems concerned in fact the majority of the presentations at this workshop, and they dealt mainly with regularisation and dual formulations. Bayesian inversion as well as applications to areas outside of finance were also considered.

  Here the lecturers were G. Bal, H. T. Banks, F. Bauer, N. Bissantz, L. Borcea, R. Cont, M. Deistler, T. Helin, B. Hofmann, Th. Hohage, S. Kindermann, B. A. Mair, P. Mathé, I. Mizera, A. Munk, J. S. Pang, S. Pereverzyev, H. K. Pikkarainen, O. Pironneau, M. Pricop, L. Roininen, S. Siltanen, K. Sutton, J. Zubelli.

– "Workshop on Computational Methods with Applications in Finance, Insurance and the Life Sciences and Stochastic Methods in Partial Differential Equations and Applications of Deterministic and Stochastic PDEs": Given that RICAM puts a strong emphasis on computational methods, it was appropriate to have a thematic workshop on such methods. It concerned direct applications to finance as well as indirect ones via applications to (stochastic) PDEs and mean field games. Methodologies ranged from multilevel Monte Carlo to optimal quantisation, to

a rough pathwise theory of nonlinear SPDEs and to various expansions, among which Fourier-cosine expansions for option pricing.

The lecturers here were Y. Achdou, V. Bally, B. Bouchard, P. Forsyth, P. Friz, S. Geiss, M. Giles, E. Gobet, B. Jourdain, A. Kohatsu-Higa, C. Oosterlee, G. Pagès, A. Pascucci, Ch. Reisinger, Ch. Schwab, J. Teichmann, E. Thomann, E. Waymire.

3.4.4 The surrounding workshops/meetings

The thematic workshops were preceded by a tutorial/autumn school on basic notions relating to the themes of the Semester, mainly for the benefit of younger participants.

The themes (in brackets the name of the lecturer) were "Optimisation Problems in Finance under Full and Partial Information" (W. Runggaldier), "Risk Modelling in Insurance" (H. Albrecher), "System Identification: General Aspects and Structure" (M. Deistler), "Pricing and Hedging under Transaction Costs" (W. Schachermayer), "Optimal Investment and Hedging under Partial Information" (M. Monoyios), "Partial Differential Equations in Option Pricing" (J.-P. Fouque), "Nonlinear Statistical Inverse Problems and Instrumental Variables" (Th. Hohage).

The thematic workshops were also preceded by a so-called "Kick-off Workshop". The latter was composed of two parts. On the first day, it was called "Practitioners Meet Academics" and involved academic consultants for the industry as well as practitioners, some from Austrian banks. For the remaining days of the week, it was a high-level workshop on all the topics of the Semester. The topics ranged from stochastic volatility modelling and utility indifference pricing for real options to default modelling and credit derivatives as well as to various aspects of portfolio choice and portfolio liquidation. Furthermore, it also concerned statistical aspects.

The lecturers for the first part were J. Crosby, A. E. Vallejo, S. Fink, P. Leoni, D. Madan, P. Schaller, A. Weingessel, and for the second D. Becherer, M. Deistler, D. Filipović, J.-P. Fouque, C. Klüppelberg, R. Korn, P. Laurence, H. Pham, Ch. Reisinger, W. Schoutens, J. Teugels, Th. Zariphopoulou, X. Y. Zhou.

Finally, there was a "Concluding Workshop". It consisted of a series of invited lectures to form the backbone of the workshop, and above all gave the possibility for some of the participants of the Semester to make an oral contribution. The contributions covered a great variety of topics that are difficult to summarise, but that allowed to get a quite clear picture of the state-of-the-art at that period.

The list of the speakers (invited lecturers and contributing participants) here is as follows: E. Alòs, M. Arisawa, C. Constantinescu, Ch. Cuchiero, S. Gerhold, V. Goldammer, Z. Grbac, M. Hahn, E. Hausenblas, F. Hubalek, P. Jaworski, M. Keller-Ressel, D. Kortschak, J. Leitner, R. Loeffen, Ph. Mayer, A. Papapantoleon, I. Penner, U. Schmock, C. Sgarra, S. Thonhauser.

A selection of contributions is published in [1], and more details can be found on the website https://www.ricam.oeaw.ac.at/specsem/sef/.

4 The Schools

4.1 Schools in Cortona

As already mentioned, the scientific institution SMI (Scuola Matematica Interuniversitaria) is regularly organising summer courses of 2–3 weeks at the Palazzone in Cortona that are addressed to students who have already been introduced to research. On three occasions, more specifically in 1997, 2006 and 2011, SMI organised a course relating to mathematics for finance and insurance. The structure of these courses, which reflects the general structure of the courses in Cortona, was such that a foreign expert would be invited to deliver the main lectures on a topic of his/her choice and would be assisted by myself as liaison to the local structures as well as to diversify the lectures. A remarkable aspect was that all participants were asked to prepare a presentation of about half an hour on a topic of their expertise; for the younger participants, this would be on the subject of their thesis. This allowed lecturers and participants to get an interesting overview of the directions of research in the various institutions that the participants came from. Many Italian and non-Italian experts in financial mathematics were participants in one of the three courses. The foreign invited lecturer of the 1997 course was Phelim Boyle, and he gave a general introduction to financial mathematics. For 2006, it was Uwe Schmock whose lectures concerned a detailed study of the first chapters of Philip Protter's book [16]. For 2011, it was Eckhard Platen who lectured on his forthcoming book with Nicola Bruti-Liberati [15].

4.2 Schools in Brixen/Bressanone

4.2.1 Venue

The two Schools in 1996 and 2003 were sponsored by the CIME foundation and took place at the Cusanus Academy in Brixen/Bressanone, a pleasant little township in the Alps, close to the famous Dolomite mountains. It had historical importance as the seat of a bishop who in addition to being a religious leader was also a secular leader. The Cusanus Academy is the property of the diocese and carries the name of one of the local bishops (1404–1464) who was known as theologian, philosopher and natural scientist. It is nicely located in the centre of the town in the middle of a park next to a river, and from the main lecture room, one gets a beautiful view of the surrounding mountains. It is mainly intended for events related to religion, but can also be hired for other purposes.

4.2.2 The first CIME School in 1996

This took place from July 8 to 13, 1996. It was suggested by Arrigo Cellina, an analyst who was a member of the CIME scientific committee, and originally I was supposed to co-organise it with Ivar Ekeland who could not continue his commitment with the School because of a major governmental appointment; it was then Jean-Michel Lasry who helped me with the invitations, in particular for what concerns the French

participation. The topics of the School, under the name of "Financial Mathematics", ranged from more mathematical to more economic ones, where also the latter were treated with mathematical formalism and rigour. They reflect to a good extent the topics that were of major interest at that moment and had a considerable impact in the sequel. The five topics are listed below, and the corresponding lecturers (in the same order as the topics) were Nicole El Karoui, Jakša Cvitanić, Tomas Björk, Elyès Jouini and Bruno Biais. The contents of the lectures were published in the CIME subseries of the *Springer Lecture Notes in Mathematics* [18]. The School was attended by over 100 participants from 16 countries, and many of them are now well-known experts in the field. Next I am shortly reviewing the contents of the lectures on the various topics:

- "Nonlinear pricing theory and backward stochastic differential equations": The main purpose of this lecture series was to show how the theory of backward stochastic differential equations (BSDEs) turns out to be a useful tool for the pricing of contingent claims. Using BSDEs, a nonlinear arbitrage pricing theory in an imperfect market can in fact be developed. In the lectures, some typical situations of imperfect markets were considered where the price system is no longer linear, and by the use of techniques from BSDE theory, various properties were shown such as consistency, absence of arbitrage and admissibility for the seller. It was also shown that the resulting price process corresponds to the upper price with respect to so-called fictitious linear markets. Finally, the problem of pricing American contingent claims in an imperfect market was considered; in the context of the lectures, this leads to reflected BSDEs.
- "Optimal trading under constraints": The topic of these lectures was the problem of hedging and utility maximisation for a single agent in a continuous-time financial market under convex constraints for the investment fractions. Different variants of the basic diffusion-type model were considered, among them the cases when the asset price dynamics depends on the trading strategy, when the interest rate for borrowing is larger than that for lending, or when there are transaction costs. The methodology was largely based on duality theory, and the mathematical tools involved continuous-time martingales, convex duality and forward-backward SDEs.
- "Interest rate theory": At the time of the School, the theory of the term structure of interest rates was lacking a comprehensive treatment of the state-of-the-art. One may well say that this comprehensiveness was achieved in this series of lectures. They concerned mostly modelling issues and the bond market, giving an overview of short rate modelling and affine term structures, as well as the at that time recent Heath–Jarrow–Morton approach. They also covered auxiliary material such as the change of numéraire technique and forward measures, and hinted at newer directions like discontinuous market models and risky bonds.
- "Market imperfections, equilibrium and arbitrage": These lectures presented topics on markets with frictions, in particular characterisations of bid and ask security price processes that are arbitrage-free, showing that such processes can be obtained as a perturbation of a price process that is arbitrage-free in a frictionless economy. Some emphasis was given to bounds on the bid–ask prices that define a possible range for the price at which a security could be traded. The interval defined by these arbitrage bounds is equal to the set of expectations of the payoff of the claim

with respect to all martingale measures, and these turn out to be the tightest bounds that one can find without the knowledge of the agents' preferences. Extensions of this framework to markets with short sales constraints and different borrowing and lending rates as well as in the presence of transaction costs were also mentioned.
– "Risk sharing, adverse selection and market structure": While the previous lectures were more on the mathematical side, this lecture series was intended to bring to the attention of mathematicians some issues that had been debated more in economics research, in particular for what concerns market microstructure and the implications of asymmetric information for frictions. The lectures were divided into two parts: a first part where there are also noise traders, and a second part where there is no noise trading and endogenous trading stems from risk sharing as well as from informational motivations. The mathematical tools built on variational calculus and convex analysis.

4.2.3 The second CIME School in 2003

This second School took place from July 6 to 12, 2003 and went under the name "Stochastic Methods in Finance" to reflect more clearly the leading methodology as it had evolved to deal with the problems arising from the financial sector. The developments in the area of financial mathematics by that time covered a wide spectrum of topics, and so the selection of the topics for the five lecture series was not an easy one. A choice had to be made and we did it by choosing as underlying theme that of "risk". Still, the methodologies ranged from the theory of stochastic processes to stochastic differential equations, to optimal and stochastic control as well as to convex analysis and duality theory. The lecturers were Walter Schachermayer, Shige Peng, Tomasz Bielecki, Christian Hipp and Kerry Back. This School was co-directed by Marco Frittelli, and in addition to being a CIME School, it was also co-sponsored by the EMS (European Mathematical Society) as one of its two Summer Schools for 2003. It was attended by 115 participants, 85 of them being Europeans. Here, too, the contents of the lectures were published in the CIME subseries of the *Springer Lecture Notes in Mathematics* [10]. Again, I am shortly reviewing the contents of the five lectures (arranged in the same order as the lecturers).

– "Utility maximisation in incomplete markets": The classical utility maximisation problem in continuous-time stochastic markets had received a renewed impulse in the mid 1980s when the so-called duality approach entered the scene. The lectures on this topic covered recent results on the duality theory in the general context of semimartingale financial market models. In order to allow one to grasp more easily the basic ideas, a first part of the lectures considered the case of a finite underlying probability space. In view of the underlying theme "risk", one may add that utility maximisation may have an impact also on asset pricing in incomplete markets; in fact, since the risk there cannot be completely hedged away, agents' preferences must in fact be taken into consideration.
– "Nonlinear expectations, nonlinear evaluations and risk measures": In this lecture series, the theory of so-called *g-expectations* was presented, thereby focusing on backward stochastic differential equations, filtration-consistent expectations,

g-martingales and decompositions of filtration-consistent supermartingales. Applications to the theory of risk measures were suggested with particular emphasis on issues of time-consistency of dynamic risk measures.
– "Modelling and valuation of credit risk": Among the different aspects of risk in finance, credit risk had received major attention in those years. This lecture series gave a broad overview of recent models and methodologies for the management of credit risk: structural models, intensity-based models, models for dependent defaults and migrations, defaultable term structures, copula-based models.
– "Stochastic control with applications in insurance": Risk has a traditional and dominating place in insurance. The series of lectures showed how the theory of stochastic control and optimisation can be used effectively and how it can be integrated with the classical insurance and risk theory.
– "Incomplete and asymmetric information in asset pricing theory": Partial and asymmetric information is doubtlessly a possible source of risk, but it has considerable importance in itself since information is generally neither complete nor equally shared among agents. The lectures presented a survey, in the context of economic equilibrium, of incomplete and asymmetric information (insider) models. Particular emphasis was given to the Kyle model and extensions thereof.

5 Some additional events/activities where I was not involved directly

5.1 Workshops in Quantitative Finance

In the late 1990s, also thanks to the initiatives mentioned above, a critical mass of scholars working in financial mathematics was by then present in Italy. Moreover, the growing number of Italian PhD students having their thesis related to this topic gave a further impulse to the need of having a dedicated "Italian" event. For this reason, in the year 2000, the first "Workshop in Mathematical Finance" took place in Pescara, organised by Fabio Antonelli, Emilio Barucci and Carlo Mari. Already with the first edition, these Italian events were characterised by the fact that they were devoted especially to young researchers, ideally PhD students and post-docs. For this reason, there were no invited speakers or submission fees, and the workshop was without parallel sessions so that everyone could hear, and be heard by, everyone. A peculiar aspect of the first editions was that they were also held in Italian since at first this was meant as a local initiative for the Italian community, even having the Italian title "Workshop in Finanza Matematica". Furthermore, the presentation of an unfinished paper was explicitly encouraged, just to exchange ideas on future research.

The first edition of 2000 was already a success, with around 40 attendees and 24 presentations. This encouraged the organisers to have the workshop as a yearly event, which actually was already planned from the beginning, with successive editions held in several Italian universities. After the first editions, still in Italian, it was clear that the workshop was gathering more and more interest, both from the Italian finance community in a broad sense, i.e., not only mathematicians but also e.g. statisticians, economists and practitioners, as well as from non-Italian scholars. For this reason, three changes were introduced in 2005 into the original formula, which

are still in force to this day. The first change was to extend a mainly Italian event, with many contributions in Italian, to become an all-international workshop, with every contribution in English. The second change concerned the name of the workshop by substituting "Mathematical" with "Quantitative", in order to enlarge the audience and the contributions to the broader community that was forming. Eventually the name became the present "Workshop in Quantitative Finance" (WQF), sometimes also written as "Quantitative Finance Workshop". The third novelty was to introduce a discussant, possibly for each presentation. This formula was quite common in many important conferences in finance, but much less in conferences in mathematics; with this novelty, everyone presenting a contribution was supposed to accept to be a potential discussant for another paper.

The above formula, which basically continues up to now, immediately found a large success, so that the one-track format was no longer sufficient and parallel sessions had to be introduced. Moreover, the international attendance became more and more significant, with several renowned scholars who are now fixed presences. Over the years, also the scientific committee grew from the initial three founding members, who are still alternating their presence in the committee, to a much wider composition now comprising about 15 members, many of which are non-Italian and/or come from the financial industry. Furthermore, WQF was often the occasion for initiatives going beyond the workshops. Examples are special issues of journals and the Young Investigators Training Program, an initiative of an Italian association of banks which provides one-month fellowships for young researchers from outside of Italy to be spent in Italian universities. In the last years, this program saw 15 such young scholars being awarded such a fellowship.

One novelty of the latest years is that WQF may also be held outside of Italy. In fact, the 20th edition (that of 2019) was held in Zurich, thanks to the suggestion of Josef Teichmann to host it (co-organised with Christa Cuchiero and the ETH financial community). This might be repeated every 5 years (thus again in 2024, 2029, etc.) by other European organisers, but the Italian community seems to be well determined to keep the event "Italian at heart", warmly welcoming contributors from abroad and keeping most of the editions in Italy.

5.2 Spring Schools in Bologna

A series of events called Spring School in Finance was organised in Bologna under the patronage of the University between 2004 and 2019. Originally, the organisers were Francesca Biagini and Andrea Pascucci; from 2007 onwards, it was only Andrea Pascucci. These Schools were intended to be addressed to a wide audience and designed for academics and researchers as well as practitioners and business people. The aim was to lead the participants to the forefront of research by providing short, intensive and up-to-date courses. The structure of the Schools was such that the courses were given by two leading researchers in a given field and consisted of eight expository lectures. The topics of the Schools were as follows: "Pricing and Risk Management of Derivative Securities" (L. Hughston) as well as "Portfolio Optimisation" (W. Runggaldier) in 2004, "Optimal Stochastic Control with Finance Applications" (T. Björk) as well as "Credit Risk Modelling and Credit Risk Management"

(Ph. Schönbucher) in 2005, "Volatility Modelling" (B. Dupire) as well as "Optimal Stopping and American Options" (D. Lamberton) in 2006, "Calibration Methods for Derivative Pricing Models" (R. Cont) as well as "Interest Rate Modelling" (F. Mercurio) in 2007, "Stochastic Modelling of Electricity Markets" (F. E. Benth) as well as "A Course on Commodities" (H. Geman) in 2008, "Lévy Driven Financial Models" (E. Eberlein) as well as "Pricing and Hedging Jump Risk" (P. Tankov) in 2009, a crash course on "Credit and Counterparty Risk" (S. Crépey and A. Pallavicini) in 2016, "XVA Modelling" (A. Pallavicini and M. Francischello) in 2019.

5.3 Further activities

In the late 1990s and early 2000s, an important role for the promotion of financial mathematics, mainly among interested Italian researchers, was played by a series of lectures at the Scuola Normale Superiore (SNS) in Pisa, mostly within the framework called "Cattedra Galileiana". This was made possible also thanks to the scientific initiative of Maurizio Pratelli from the University of Pisa, who is one of the early Italian probabilists having become interested in financial mathematics. Many well-known scientists were among the lecturers, such as (in alphabetic order) M. Avellaneda, T. Björk, H. Bühlmann, F. Delbaen, P. Embrechts, N. El Karoui, D. Kramkov, H. Pham, W. Schachermayer, M. Schweizer, M. Soner. Some of the lectures were published in an internal series of SNS, the "Collana Quaderni Cattedra Galileiana".

With the increased interest in financial mathematics among Italian probabilists, some PhD programs in the field were started also in mathematics departments in addition to some existing ones in departments of economics. Among the first such programs, there was one at the Scuola Normale (SNS), with theses mostly supervised by Maurizio Pratelli, and one at the mathematics department of the University of Padova. In the sequel, analogous programs were also started in other major Italian universities. To conclude, I should like to mention that also various programs at the Master level (professionalising Masters) were introduced in Italy. The early ones were at departments of economics, but then they were activated also upon initiatives of mathematicians. The first such more mathematically oriented program appears to be the successful "Alta Formazione in Finanza Matematica" at the University of Bologna; it was started in 2003 upon the initiative of Andrea Pascucci and Sergio Polidoro and has now arrived at its 17th edition with an average of 25 students per year. Analogous programs have then been implemented also in other Italian universities.

6 Concluding remarks

The period of reference of this survey, namely between 1992 and 2008, is when I was active in the organisation of events in financial mathematics, of which I have considered here the major ones. As already mentioned, this period has some relevance in itself. Indeed, around 1992, the field that we now consider under the name of financial mathematics was in the coming as a field by itself. The ending year 2008 roughly marked the beginning of the so-called big financial crisis that lasted for about four years and during which some major changes took place in the financial industry, without, however, affecting crucially the basics of financial mathematics.

To substantiate the statement above that 1992 was a period when the field started to grow, I should like to describe in the next subsections how everything came about. This may then also allow readers to get a feeling of where many of the original experts in financial mathematics came from and how they started to work in the field. In the two subsections to follow, I try to summarise some of the main points of the evolution as described in this survey. The first of these further subsections concerns the topics, the second one the mathematical methodologies.

6.1 How things originated

To be able to organise a major conference and to attract relevant presenters, one needs above all to have good connections. This is especially important in the beginning in order to reach a critical mass of speakers that will then by itself attract further relevant people. Another important aspect is to have an attractive venue and sufficient funding. For this latter aspect, I was fortunate enough to have good connections with Franco Giannessi, a specialist in optimisation from the University of Pisa who at the Erice centre is/was in charge of the mathematics school that is named after Guido Stampacchia, an Italian mathematician who became known through his work in variational analysis, in particular variational inequalities. This is how I was able to organise my first event in Erice. Concerning the lecturers not only of the Erice conference, but also of the following ones, I should like to mention three major "connections" that I may call the American, the French and the Russian connection. These were possible because next to my affiliation with the Mathematics Department in Padova, I had in my early days a partial affiliation also with a laboratory of the Italian National Research Council (CNR) which at that time was called LADSEB. This lab had a strong group working in (stochastic) dynamical systems with good connections abroad. Through one of these connections, I had the opportunity of visiting twice, in the late 1970s and mid-1980s, the Laboratory for Information and Decision Systems at MIT. During the second of these visits, also Yannis Karatzas and Steven Shreve were there. They were on leave from their institutions and were working on their book "Brownian Motion and Stochastic Calculus". I had then the excellent opportunity to attend a course on stochastic analysis that Dan Stroock was giving to just four of us, Yannis, Steven, José Moura (now at CMU) and myself. This led to what I call the American connection, and in fact, Yannis was one of the lecturers in Erice.

The connection with France came also via LADSEB in the person of Alain Bensoussan whom I had been visiting at INRIA in the second half of 1984. It was during this period that I came in contact with Nicole El Karoui and her group. Also via LADSEB I had already previously established some contacts with Monique Jeanblanc-Picqué who also came to Erice.

Concerning the Russian connection, I was able to organise together with my colleague Gianni Di Masi a meeting at LADSEB in 1985 in which participated some famous Russian probabilists, among them N. Krylov, R. S. Liptser and A. Veretennikov. In the days of the Soviet Union, it was really difficult to keep contacts with Russian scientists, but together with Gianni, we even succeeded to have in 1987 a visiting position at IIASA (International Institute of Applied System Analysis) in Vienna. The invitation came from Alexander B. Kurzhanski, the chairman of the System and Decision Sciences Program there; he was aware of our activity in the field of

stochastic filtering and control and I was known to have a fair knowledge of the Russian language. During the so-called Cold War, IIASA was basically the only scientific institution where Westerners and Soviets collaborated; our project leader was in fact a Russian scientist who later became a member of the Russian Academy of Sciences. It was there that our contacts with Yuri Kabanov originated. He then participated in most of the conferences and I had the chance to collaborate rather extensively with him in financial mathematics. The first and perhaps most successful work with him came about as follows. At LADSEB, my colleague Andrea Gombani (in Italy, Andrea is a male name), who holds a PhD in systems theory from KTH in Stockholm, had invited Tomas Björk to work on stochastic filtering. At the same time, Gianni Di Masi had invited Yuri Kabanov to work on two-scale systems. It turned out that both Björk and Kabanov had started to get interested in stochastic methods in finance, and so we joined forces to work together on a topic that at the time could have been of interest. Upon a suggestion by Yuri, we then started working on the term structure of interest rates, which led to the two papers [3, 5]. Andrea, too, then started working in financial mathematics; see e.g. Björk and Gombani [4].

This concludes the three major "connections". There were others like what I could call the "German connection" that was quite natural for me, having grown up with the German language and also because I had spent four years as an assistant professor at the University of Zürich in what was eventually called "Institute for Operations Research and Electronic Data Processing". All these connections made it certainly easier for me to invite people, but I was also very lucky that people were kind enough to accept the invitations, and for this I am grateful to them.

6.2 On the topics

There are some issues that are recurrent in financial mathematics, as one can see also by looking through the topics of the various conferences.

6.2.1 Economic/financial principles

The theoretical/mathematical development of financial mathematics needs some economic/financial principles upon which to build. This is essentially the equilibrium in financial markets and the principle of absence of arbitrage. More than an explicit topic by itself, this was underlying many of the presentations. There were, however, some more explicit presentations in this context as e.g. the talk "Arbitrage theory" by F. Delbaen in Trento, the lecture series "Market imperfections, equilibrium and arbitrage" by E. Jouini at the 1996 Summer School in Brixen/Bressanone, where market frictions were taken into account, and "Asymptotic arbitrage in large financial markets" by Yu. Kabanov in the 1994 Cortona conference. Another issue that can be seen in this context is that of asymmetric or insider information. This was implicitly present in some of the talks, but was explicitly dealt with in the lecture series "Incomplete and asymmetric information in asset pricing theory" by K. Back in the 2003 Summer School in Brixen/Bressanone. Insider information became a topic of major interest in the years after the period we are considering here, and this also in relation to weaker forms of absence of arbitrage. In fact, although it was in the coming,

there were not yet any presentations concerning directly weaker forms of absence of arbitrage beyond NFLVR, or of the so-called *benchmark approach* for which the physical measure can be taken as a pricing measure.

6.2.2 Modelling of price evolution

This, and the related topic of price formation, is one of the early topics going back to Bachelier himself. Various price evolution models beyond the classical Black–Scholes model were discussed throughout the events. For the period we are considering here, it was still early for price impact models; nevertheless, there was a presentation by M. Schweizer at the 1994 Cortona conference on "Stock price models with feedback effect". Some presentations concerned the market microstructure, as e.g. those by H. Föllmer at the Erice and the 1994 Cortona conferences. Rough path theory had not yet the interest that it attracted later; there was, however, a presentation by P. Friz at the Special Semester in Linz in 2008 on "Towards a (rough) pathwise theory of fully nonlinear PDEs".

6.2.3 Pricing and hedging contingent claims

This is an omnipresent topic in financial mathematics, and it also was so for the events described in this survey. In the beginning of the period considered here, there was great interest in pricing and hedging approaches in incomplete markets, and various presentations were related to the quadratic hedging approach according to Föllmer and Schweizer that had just been worked out as well as to various specific martingale measures. The ensuing local risk minimisation approach was discussed in the context of various applications such as when there are transaction costs, for American options, etc. Then, always for incomplete markets, there came utility indifference pricing, first presented by M. Davis at the 1994 Cortona conference. A successful tool in this context turned out to be the theory of backward stochastic differential equations, on which N. El Karoui had lectured during the 1996 Summer School in Brixen/Bressanone. Related to this is the theory of g-expectations on which S. Peng had lectured during the 2003 Summer School, always in Brixen/Bressanone. Hedging and pricing when portfolios are constrained to take values in a closed convex set was a subject already in one of the presentations of Y. Karatzas at the 1992 Erice conference.

6.2.4 Portfolio optimisation

This is among the original and basic problems in financial mathematics. Major emphasis was given to this topic during the 1992 Erice conference, where Y. Karatzas presented his at the time recent work with J. Cvitanić on the convex duality approach in constrained portfolio optimisation. This was then picked up and generalised in the lecture series on "Optimal trading under constraints" by J. Cvitanić at the 1996 Summer School in Brixen/Bressanone. Various other issues in portfolio optimisation that attracted interest also beyond 2008 were touched upon during the events described here, such as optimisation with transaction costs, optimal portfolio liquidation, risk sensitive asset management, stochastic target problems, and investment

under drawdown constraints. An extensive overview on the duality approach in the general context of semimartingale models was given by W. Schachermayer during the 2003 Summer School in Brixen/Bressanone. Major emphasis was also given to optimal strategies for insurers like in the 1998 Cortona conference which was focused on the relationship between finance and insurance, as well as during the 2003 Summer School in Brixen/Bressanone.

6.3 On the mathematical methodologies

Here I want to limit myself to summarising some of the more general mathematical models and methodologies that were underlying the various presentations.

6.3.1 Modelling

Continuous and discontinuous semimartingale models, stochastic volatility and regime-switching models, models with random trading times, modelling of extremal events, intensity-based models for defaultable term structures, modelling spot and forward prices in energy markets, differential geometric approaches to obtain finite-dimensional realisations of interest rate models, PDEs in option pricing. There were not (yet) presentations concerning models for high-frequency data.

6.3.2 Measures and information structures

Change of measures and change of numéraire, forward measures, Esscher transforms, enlargement and shrinking of filtrations (asymmetric and incomplete information structures), nonlinear expectations (g-expectations).

6.3.3 Analytical tools

Forward-backward SDEs, also with reflection and driven by a Poisson random measure, variational properties of BSDEs, numerical methods for SPDEs and BSDEs, backward SDEs with constrained jumps, and quasi-variational inequalities. There were also some applications of large deviations, in particular for asymptotic arbitrage, and one presentation concerning nonstandard analysis in option pricing.

6.3.4 Optimisation

During the events described here, this relates mainly to portfolios, namely convex duality, dual formulation of target problems, variational calculus, portfolio turnpike theorems, risk sensitive control, optimal control under partial information, portfolio optimisation in pure jump markets, behavioural models under prospect theory, coherent and dynamic risk measures.

6.3.5 Approximations

Monte Carlo simulation methodologies, expansions, approximations based on discretisation of SDEs, also with jumps (in view of pricing American options and solving BSDEs), optimal quantisation.

6.3.6 Further methodologies

Statistical estimation of volatility, inverse problems methodologies.

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