Review on *Steps toward a dynamic costrucivism* by John L. Bell

Traditional explications of the nature of mathematics have tended, broadly speaking, to assume one of two forms. In the first, the primary datum is taken to be the seemingly *objective* character of mathematics, namely the fact, assented to in one way or another by every mathematician and first given systematic articulation by Plato, that its contents are in some sense objectively *true* or *correct.* Adherents of the second form follow Kant in taking the essence of mathematics to be the generation of mental *certainty*—that, in the end, mathematical knowledge is knowledge of the human mind, or, in the last analysis, self-knowledge. In a word, ontology—or epistemology? Realism, or idealism?

In outlining what he calls *dynamic constructivism* the author of the present paper proposes an approach to the problem which, while containing idealist and realist elements, is irreducible to either. He draws a parallel between his approach and developments in modern biology:

> "Many novel ideas have come from biology, chiefly since the discovery of DNA in the 1950s. Of the greatest interest here is the fact that… it is now generally believed by biologists and neuroscientists that nothing beyond biology and evolution is theoretically necessary to explain human beings, that is, their bodies and their minds. … My general claim is that simply that the same holds for logic and mathematics. … That is, mathematics is a product of our minds and so to explain it we require no more than what is needed by biologists to explain the mind."

This claim is, at first glance, very similar to that made by Lakoff and Nunez in their book *Where Mathematics Comes From.* But actually the author is not claiming that mathematics is reducible to biology or to anything else; rather, he is making an analogy between *evolutionary* biology in which organisms, including brains, develop over time in interaction with their environment and mathematical concepts which are continually undergoing a similar process in theirs. The aim is to see "everything from a dynamic, rather than a static, viewpoint."

The author's starting point is Brouwer's intuitionism, the idealist contention that mathematics consists in the mental constructions of an individual. While Brouwer's philosophy of mathematics was rightly "rejected by most because of its links with mysticism", the author maintains that "Brouwer's main claim, namely that no mathematical object or truth is given and that all must be constructed is … correct." Indeed, he believes that "most of Brouwer's insights are correct." To obtain dynamic constructivism, "in the last analysis…it is enough to drop Brouwer's justification for the intersubjectivity of mathematics, which is essentially the direct mystical intuition of the creative subject, and simply replace it with the existence of other individuals." That is, one might say, to open out the hermetic world of the subject into the *umwelt,* the lived,

changing world which includes other individuals, their perceptions and the interactions both between themselves and the natural environment. The whole mathematical universe exists in this *umwelt*, "in the minds of human beings". The objectivity of mathematics then amounts essentially to a shared process of reification, by a "democratic, though occasionally turbulent, dynamic process of achieving consensus." Such a process must include "the interaction with other individuals, and with the outcome of *their* mental processes."

Dynamic constructivism accordingly has its roots in idealism, but it can also be related, as the author observes, to realism (or Platonism). This may be achieved by discarding "the static view, the [assumption] that concepts, or ideas, are statically given somewhere." Or inversely, one may "think of Platonism as obtained from dynamic constructivism by postulating that the process of construction of a concept actually has an aim, a télos, so that all our trials and errors are just approximations converging to something that exists already." In a pragmatic spirit, it is suggested that we "do without the useless, troublesome assumption of the convergence point, and accept that there is *only* the process of trial and error."

The author contrasts dynamic constructivism with *formalism* in mathematics, whose apparent aim is "to reach the certitude of objectivity by cutting off whatever involves a subject, and so all intentions, intuitions, meanings, etc." Formalism is unsatisfactory since it "fails to explain the meaning of mathematics and how it is done"; it is also illusory, for it "seems to ignore the fact that as soon as a language…is considered, a subject reading that language… is automatically on the scene." It is suggested that the subject be reintroduced by restoring the role of the metalanguage. In general, as the author sagely observes, "the one safe path to objectivity is to *transcend* the subject; pretending that there is no subject is a false shortcut, which in the end is similar to Brouwer's pretence that there is only one."

That is, formalism does away with the subject altogether, intuitionism allows just one subject, but dynamic constructivism requires the interaction of many subjects.

In §5 of the paper the author sketches some of the consequences of dynamic constructivism for logic. He observes that since "Brouwer's revolution…has introduced plurality into the space of logics", either logical pluralism has to be accepted or solid arguments have to found to justify the identification and acceptance of the one true, "ultimate" logic. The existence of the latter is essentially precluded if one understands "truth…as constructed, and hence also relative to a degree of abstraction". Upon this follows logical pluralism, and so also a pluralistic attitude towards the concept of truth itself. Just how, then, is a logic constructed? In answer to this question the author sketches the system developed by himself and his collaborators called *Basic Logic*, a sequent calculus in which all the rules of inference governing the logical connectives are derived from a metalinguistic analysis of meaning. Thus does "the dynamic relationship between metalanguage and language provide a common structure…common to all logics…deeper than any specific notion of truth."

The paper's penultimate section contains an outline of the *Basic Picture*, the account of topology and continuity developed by the author and which he sees as closely linked to dynamic constructivism.

In his concluding remarks the author observes that "the possibility of new discoveries" is stimulated by a "freer, less dogmatic frame of mind, or foundation". This path-breaking, engaging paper will, I believe, assist in the latter's emergence (long overdue) among workers in the foundations of mathematics.