University of Houston

COSC 3320: Algorithms and Data Structures
Spring 2017

Homework 3

Due February 23, at the start of class

1. Consider the following recurrence relation.

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/8) + \sqrt[3]{n} & \text{if } n = 8^d, d > 0. \end{cases}$$

   (a) Apply the Master Theorem to have an asymptotic bound for $T(n)$.
   (b) Determine the exact value of $T(n)$ using the unfolding technique.
   (c) Prove by induction the correctness of the above solution.
   (d) Verify that the above solution is coherent with the asymptotic bound obtained in (a).

2. Consider the following recurrence relation.

$$T(n) = \begin{cases} 4 & \text{if } n = 1, \\ 6T(n/3) + n(n-1) & \text{if } n = 3^d, d > 0. \end{cases}$$

   (a) Draw a table that summarizes all the relevant information obtained from the recursion tree, namely, for each level, the number of nodes, the cost of each node, and the total cost associated to the level.
   (b) From (a), determine the exact value of $T(n)$.

3. Consider the following recurrence relation.

$$T(n) = \begin{cases} n & \text{if } n \in \{1, 2\}, \\ T(n/2) + T(n/4) + n & \text{if } n = 2^d, d > 1. \end{cases}$$

   (a) Draw the recursion tree for $n = 32$.
   (b) Determine the number ℓ of levels of the recursion tree, and an upper bound to the total cost associated to level i, with $0 \leq i < \ell$.
   (c) From (b), determine an upper bound to $T(n)$, and argue that this bound is asymptotically tight.

4. Let $S = S[0], S[1], \ldots, S[n-1]$ be a sequence of n distinct elements on which a total order relation is defined. We say that two elements $S[i]$ and $S[j]$ in S are a *friendly pair* if $S[i] < S[j]$ and $i < j$. Give a recursive algorithm that determines the number of friendly pairs in S in time $O(n \log n)$. (Hint: adapt the Merge-Sort strategy.)