University of Houston

COSC 3320: Algorithms and Data Structures
Summer 2015

Homework 1

Due June 10, at the start of class

1. (a) Prove that the sum of the first n odd natural numbers is n^2.
   (b) Prove that for every integer $n \geq 0$, $\sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.
   (c) Prove that for every integer $n \geq 0$, $\sum_{i=0}^{n} i \cdot 2^i = (n-1)2^{n+1} + 2$.
   (d) Prove that for every integer $n \geq 7$, $n^2 \geq 6n + 7$.

2. (a) Prove that the function $f(n) = 8n + 5$ is $O(n)$.
   (b) Prove that the function $f(n) = 2^{n+2}$ is $\Theta(2^n)$.
   (c) Prove that the function $f(n) = 3n^3 + 4n^{5/3} + 2\log n + 8$ is $O(n^3)$.
   (d) Prove that the function $f(n) = na^{\log n}$, where a is a constant greater than 1, is $\Theta(n^c)$ for some constant c.
   (e) Prove that the function $f(n) = n^{1/\log n}$ is $O(1)$.
   (f) Prove that for any constant $a > 0$, $\log n$ is $o(n^a)$.
   (g) Order the following functions by order of growth, that is, find an arrangement $f_1, f_2, \ldots, f_{20}$ of the functions such that $f_1 = O(f_2), f_2 = O(f_3), \ldots, f_{19} = O(f_{20})$. (Here $\log n$ means $\log_2 n$.)

n^2	$\frac{1}{\log n}$	$n^{4/5}$	1.5^n	$\frac{2^{\log n}}{2}$
$n \log\log n$	$\sqrt{\log n}$	$\log^* n$	14	$\log\log\log n$
$\sqrt{n^5}$	$\log^{11/6} n$	$e^{\sqrt{n}}$	$\log(\log n^3)$	$\log(n!)$
$2^{\sqrt{\log n}}$	$\frac{n}{\log n}$	$\log\left(\frac{n}{\log n}\right)$	$\frac{\log n}{n}$	$n!$

3. The *element distinctness* problem is the problem of determining whether all the n elements of a list are distinct.
   (a) Write the pseudocode for the straightforward algorithm that tests each of the n elements for distinctness, and determine its complexity.
   (b) Give a more efficient algorithm (i.e., write its pseudocode), and determine its complexity.