University of Houston

COSC 3320: Algorithms and Data Structures
Summer 2015

Homework 5

Due July 9, at the start of class

1. Insert, in this order, the following entries in an initially empty binary search tree: $(9, x), (4, a), (17, f), (1, c), (8, a), (14, k), (20, d), (2, p), (13, w)$. You are to draw the final binary search tree.

2. Let T be a binary search tree which implements a dictionary. Let v be a node of T, and T_v be the subtree rooted at v. Design a recursive algorithm `CountLE`(v, k) which, given an input node v and a key k, returns the number of entries in T_v with key at most k.

3. Design and analyze a simple and efficient non-recursive algorithm to determine the height of a $(2, 4)$-tree.

4. Let T be a $(2, 4)$-tree containing n entries with distinct, integer keys. Suppose every node $v \in T$ maintains a variable $v.size$ that stores the number of entries contained in the subtree rooted at v (denoted T_v), included the entries in v. Design a recursive algorithm `Count` which, given an integer k, returns in $O(\log n)$ time the number of entries in T with key less than k.

5. Let $G = (V, E)$ be an undirected graph with n vertices and m edges. Prove the following properties.

   (a) If G is connected, then $m \geq n - 1$.

   (b) If G is a tree, then $m = n - 1$.