University of Houston

COSC 3320: Algorithms and Data Structures
Summer 2015

Homework 6

Due July 16, at the start of class

1. Consider the following graph, represented by its adjacency matrix.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Draw the graph.

   (b) Run the DFS algorithm starting from vertex 1, and draw the final DFS tree.

   (c) Run the BFS algorithm starting from vertex 1, and draw the final BFS tree.

2. Let $G = (V, E)$ be a connected graph where the degree of each node is c, where $c > 2$ is an integer constant. Consider the execution of the BFS algorithm, BFS(G, v), from a node $v \in V$. Prove by induction on i that the level L_i generated by BFS(G, v) contains at most c^i nodes, for each $i \geq 0$.

3. Let $G = (V, E)$ be a graph with n nodes and m edges. Design and analyze an algorithm that returns, if it exists, a node $i \in V$ such that at least $n/2$ different nodes are reachable, via a path, from i. (Hint: Use the BFS algorithm, with a suitable modification.)

4. Consider the following weighted graph, represented by its adjacency matrix.

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 4 & 1 \\ 3 & 0 & 10 & 0 & 0 & 0 & 4 \\ 0 & 10 & 0 & 7 & 0 & 0 & 8 \\ 0 & 0 & 7 & 0 & 6 & 0 & 5 \\ 0 & 0 & 0 & 6 & 0 & 5 & 4 \\ 4 & 0 & 0 & 0 & 5 & 0 & 2 \\ 1 & 4 & 8 & 5 & 4 & 2 & 0 \end{bmatrix}$$

   List the edges of the minimum spanning tree in the order they are added by Kruskal's algorithm.