Partial exam: Mathematical Methods, part A, 06/05/13

Name ....

Exercise 1 [8 points]

State and prove Liouville Theorem.

Exercise 2 [12 points]

(i) Classify the singularities of the function of complex variable

$$f(z) = \frac{e^{2iz}(e^{iz} - 1)}{z(z^2 + 4)^2}.$$

(ii) Find the singular part of f in a ball $B(c, r]$ contained in the domain, where c is a singularity and $\operatorname{Im} c > 0$.
(iii) Prove that $g(x) = \frac{\sin 3x - \sin 2x}{x(x^2+4)^2} \in L^1(\mathbb{R})$.
(iv) Compute the integral

$$\int_{-\infty}^{+\infty} \frac{\sin 3x - \sin 2x}{x(x^2 + 4)^2}\, dx.$$

Exercise 3 [8 points]

(i) Prove that the function

$$u(x, y) = \sin(\pi x) e^{-\pi y}, \quad (x, y) \in \mathbb{R}^2$$

is harmonic.
(ii) Find an harmonic conjugate of u.
(iii) Which is the underlying holomorphic function?

Exercise 4 [4 points]

Classify the singularity at 0 of the function of complex variable

$$f(z) = \frac{e^{1/z}}{\sin z}.$$