

# Representations of Algebraic Groups: A Conference in Honour of Stephen Donkin

13th - 15th July 2016, York

## Titles and Abstracts

### Wednesday 13th July

**Richard Dipper** (University of Stuttgart)

*Orbit Method for  $p$ -Sylow Subgroups of Finite Groups of Lie Type*

**Abstract:** This is joint work with Q. Guo, M. Jedlitschky, Y. Sun and M. Werth.

Based on Kirillov's orbit method C.A.M. Andr e and later, but independently, N. Yan constructed a supercharacter theory for the finite unitriangular groups  $U_n(a) \in \text{Syl}_p(\text{GL}_n(q))$ , where  $q$  is a power of some prime  $p$ . More recently C.A.M. Andr e and A.M. Neto extended this to the  $p$ -Sylow subgroups  $P(q)$  of finite classical groups of untwisted type by restricting supercharacters to and intersecting superclasses of the overlying  $U_n(q)$  with  $P(q)$  obtaining in this way supercharacter theories for these groups. In his doctoral thesis M. Jedlitschky developed a general method, called monomial linearisation, to adapt the original orbit method to the groups  $P(q)$  directly. As a result he obtained for the  $p$ -Sylow subgroups of the even orthogonal groups  $O_{2n}^+(q)$  a further decomposition of the Andr e-Neto supercharacters into characters afforded by transitive monomial representations, such that every irreducible character of  $P(q)$  is constituent of precisely one of those. In this lecture I present a slight generalisation of Jedlitschky's monomial linearisation and some recent applications obtained by my working group.

**William Crawley-Boevey** (University of Leeds)

*On quiver Grassmannians and orbit closures for representation-finite algebras*

**Abstract:** This is joint work with Julia Sauter. We show that Auslander algebras have a unique tilting and cotilting module which is generated and cogenerated by a projective-injective; its endomorphism ring is called the projective quotient algebra.

For any representation-finite algebra, we use the projective quotient algebra to construct desingularizations of quiver Grassmannians, orbit closures in representation varieties, and their desingularizations. This generalizes results of Cerulli Irelli, Feigin and Reineke.

**Karin Erdmann** (University of Oxford)

*$\Delta$ -finite quasi-hereditary algebras*

**Abstract:** Let  $A$  be a quasi-hereditary algebra, and let  $\mathcal{F}(\Delta)$  be the category of modules which have a filtration by standard modules. Then  $\mathcal{F}(\Delta)$  is closed under direct summands, and one would like to understand the indecomposable modules in this category, such as when the category has finite type. One can use Auslander-Reiten theory, or, if one is lucky, information on ext spaces. We discuss some results and some questions, mostly for the case

when  $A$  is a Schur algebra. [Some of this is joint with D. Madsen and V. Miemietz, and also work with A. Cox.]

**Henning Andersen** (Aarhus University)

*Two open problems in modular representation theory*

**Abstract:** Consider a connected reductive algebraic group  $G$  over a field of characteristic  $p > 0$  and let  $G_1$  denote the kernel of the Frobenius homomorphism  $F : G \rightarrow G$ . In this talk I will discuss the following two questions and their relations to recent developments.

1. Do indecomposable injective  $G_1$ -modules always have  $G$ -structures?
2. How can we compute the coherent sheaf cohomology of vector bundles on the flag variety for  $G$ ?

**Thursday 14th July**

**Maud De Visscher** (City University)

*Representations of the partition algebra*

**Abstract:** The partition algebra was introduced by Paul Martin in the context of statistical mechanics. In this talk I will explain how it can be used to give a new approach to the Kronecker problem. The Kronecker coefficients describe the decomposition of the tensor product of two Specht modules for the symmetric group. Finding a positive combinatorial formula for these coefficients is wide open in general; this is known as the Kronecker problem. The partition algebra satisfies a double centraliser property with the symmetric group. Using this, we obtain a new interpretation of the Kronecker coefficients in the context of the partition algebra. I will explain this in details and give some applications. This talk is based on joint works with C. Bowman, J. Enyang and R. Orellana.

**Anton Cox** (City University)

*Kazhdan-Lusztig polynomials and diagram algebras*

**Abstract:** The representation theory of diagram algebras can often be described using Kazhdan-Lusztig polynomials. In this talk we will briefly review how this applies to “traditionally” defined diagram algebras, before considering recent work with Chris Bowman and Liron Speyer on diagrammatic Cherednik algebras.

**Stephen Doty** (Loyola University)

*Schur-Weyl duality for the free Lie algebra*

**Abstract:** (Joint work with Matt Douglass.) Classical Schur-Weyl duality relates the commuting actions of the general linear group  $GL(n)$  of  $n \times n$  matrices and the symmetric group  $Sym(r)$  on  $r$  letters, acting on the  $r$ -th tensor power of the natural  $n$ -dimensional vector space  $V$ , by the statement that the invariant endomorphisms for each set of operators is actually fully generated by the image of the other.

If we replace the tensor space in this picture by the degree  $r$  part of the free Lie algebra on the space  $V$ , we still have an action by  $\mathrm{GL}(n)$  but the symmetric group algebra must be replaced by a smaller sub-algebra defined by a Lie idempotent. Then we obtain an analogue of classical Schur-Weyl duality for this new setting, at least in the semisimple case. Half of this result holds in arbitrary characteristic, and we conjecture that the other half does, too.

**Alison Parker** (University of Leeds)

*Some central idempotents for the Brauer Algebra*

**Abstract:** The Brauer algebra is an important algebra in representation theory which is closely connected to the symmetric group. In this seminar I will define the Brauer algebra and very briefly discuss some of its basic representation theory. One method for understanding an algebra is to find central idempotents. We provide a new method for constructing central idempotents in the Brauer algebra relating the splitting of certain short exact sequences. This is joint work with Oliver King and Paul Martin.

**Jens Carsten Jantzen** (Aarhus University)

*Maximal Weight Composition Factors for Weyl Modules*

**Abstract:** Fix an irreducible (finite) root system  $R$  and a choice of positive roots. For any algebraically closed field  $k$  consider the almost simple, simply connected algebraic group  $G_k$  over  $k$  with root system  $R$ . One associates to any dominant weight  $\lambda$  for  $R$  two  $G_k$ -modules with highest weight  $\lambda$ , the Weyl module  $V(\lambda)_k$  and its simple quotient  $L(\lambda)_k$ . Let  $\lambda$  and  $\mu$  be dominant weights with  $\mu < \lambda$  such that  $\mu$  is maximal with this property. Garibaldi, Guralnick, and Nakano have asked under which condition there exists  $k$  such that  $L(\mu)_k$  is a composition factor of  $V(\lambda)_k$ , and they exhibit an example in type  $E_8$  where this is not the case. The purpose of my talk is to show that their example is the only one. I give two proofs for this fact, one that uses a classification of the possible pairs  $(\lambda, \mu)$ , and another one without case-by-case considerations.

**Friday 15th July**

**Donna Testerman** (EPFL)

*A new family of non maximal reductive subgroups*

**Abstract:** In 1987, Seitz proved a classification theorem for the maximal closed connected subgroups of the simple classical type algebraic groups defined over an algebraically closed field of positive characteristic, generalizing the work of Dynkin which covered the characteristic zero case. The majority of the maximal subgroups arise as images of irreducible representations of simple algebraic groups. Here we report on a family of irreducible representations of orthogonal groups whose image we have recently shown to be non maximal. This is joint work with Mikael Cavallin.

**Ana Paula Santana** (University of Coimbra)

*A preaction of the Hecke monoid on rational modules for Borel subgroups of quantum general linear groups*

**Abstract:** I will construct a preaction of the Hecke monoid on the category of rational modules for the quantum negative Borel subgroup of the quantum general linear group. I will also show that this preaction induces a preaction on the category of modules for the quantised Borel-Schur algebra. This is joint work with I. Yudin.

**Wilberd Van der Kallen** (Utrecht University)

*Bifunctors and Formality*

**Abstract:** If  $V$  is a finite dimensional vector space and  $B$  is a strict polynomial bifunctor, then  $B(V, V)$  is a representation of the algebraic group  $GL(V)$ . In characteristic  $p$  the  $p$ -power map defines a Frobenius twist for bifunctors and representations. We discuss the interaction between Frobenius twist and cohomology for bifunctors/representations.