ALGEBRAIC GEOMETRY 1

Study of solution sets of systems of polynomial equations

\[
\begin{align*}
F_1(x_1, \ldots, x_n) &= 0 \\
\vdots \\
F_r(x_1, \ldots, x_n) &= 0
\end{align*}
\]

\(F_1, \ldots, F_r\) : polynomials with coeff. in a field \(k\)

Easiest case: \(\deg F_1 = \cdots = \deg F_r = 1\)

\(\implies\) linear algebra

Solution set is an affine (linear) subspace of \(\mathbb{k}^n\)

Only invariant: dimension

The situation is much more mysterious if the degrees are higher.

E.g. \(r=3, n=3, \ deg F_1 = \deg F_2 = \deg F_3 = 2\)

\(k = \mathbb{C}\)

\(\{F_1, F_2, F_3\}\) are linearly independent
Ex. 1 \[
\begin{align*}
&\begin{cases} x_1^2 + x_2^2 - x_3^2 - 1 = 0 \\
x_1^2 - x_2^2 + x_3^2 - 1 = 0 \quad \implies x_1^2 = x_2^2 = x_3^2 = 1 \\
x_1^2 - x_2^2 - x_3^2 - 1 = 0
\end{cases} \\
\end{align*}
\]
Solution set: \(\{ (\pm 1, \pm 1, \pm 1) \}\) 8 points

Ex. 2 \[
\begin{align*}
&\begin{cases} x_1^2 - x_2 = 0 \quad \implies x_2 = x_1^2 \\
x_1 x_2 - x_3 = 0 \quad \implies x_3 = x_1^3 \\
x_1 x_3 - x_2^2 = 0 \quad \implies x_4^4 - x_4^4 = 0
\end{cases} \\
\end{align*}
\]
Solution set: \(\{ (t, t^2, t^3) \mid t \in k \}\) a curve

This is a foundational course
Aim: introduction to the key concepts of modern algebraic geometry
- affine and projective varieties
- sheaves
- schemes

Prerequisites: basics of commutative algebra
(can give additional references)

PLEASE send me an email if you haven’t attended any CA courses
References: Gathmann’s notes 2002/2014
Available on-line (see link on Moodle page)

Exercise sheets • available weekly
• use the forum on Moodle if you have questions on
  the exercises
• discussed during lectures

Exam: written exam

Moodle page:
• lectures + handouts
• notes
• literature
• Exercise sheets
• Forum for discussion on course material

Contact: www.math.unipd.it/~tommasi

Schedule: 32 lectures