ALGEBRAIC GEOMETRY 1

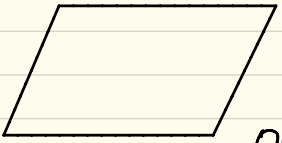
Study of solution sets of systems of polynomial equations

$$\begin{cases} F_1(x_1, \dots, x_n) = 0 \\ \vdots \\ F_r(x_1, \dots, x_n) = 0 \end{cases}$$

$F_1, \dots, F_r$: polynomials with coeff. in a field k

Easiest case : $\deg F_1 = \dots = \deg F_r = 1$

$\rightsquigarrow$ linear algebra



Solution set is an affine (linear) subspace of k^n

Only invariant : dimension

The situation is much more mysterious if the degrees are higher.

E.g. $r=3$, $n=3$, $\deg F_1 = \deg F_2 = \deg F_3 = 2$

$k = \mathbb{C}$

$\{F_1, F_2, F_3\}$ are linearly independent

Ex. 1 $\begin{cases} x_1^2 + x_2^2 - x_3^2 - 1 = 0 \\ x_1^2 - x_2^2 + x_3^2 - 1 = 0 \\ x_1^2 - x_2^2 - x_3^2 - 1 = 0 \end{cases} \rightsquigarrow x_1^2 = x_2^2 = x_3^2 = 1$

Solution set: $\{(\pm 1, \pm 1, \pm 1)\}$ 8 points

Ex. 2 $\begin{cases} x_1^2 - x_2 = 0 \rightsquigarrow x_2 = x_1^2 \\ x_1 x_2 - x_3 = 0 \rightsquigarrow x_3 = x_1^3 \\ x_1 x_3 - x_2^2 = 0 \rightsquigarrow x_1^4 - x_1^4 = 0 \end{cases}$

Solution set: $\{(t, t^2, t^3) \mid t \in k\}$ a curve

This is a foundational course
Aim: introduction to the key concepts of modern algebraic geometry

- affine and projective varieties
- sheaves
- schemes

Prerequisites: basics of commutative algebra
(can give additional references)
PLEASE send me an email if you haven't attended any CA courses

References: Gathmann's notes 2002/2014
Available on-line (see link on Moodle page)

Exercise sheets
- available weekly
- use the forum on Moodle if you have questions on the exercises
- discussed during lectures

Exam: written exam

Moodle page:
- lectures + handouts
- notes
- literature
- Exercise sheets
- Forum for discussion on course material

Contact: www.math.unipd.it/~tommasi

Schedule: 32 lectures