

Pairwise disjoint segments in the real plane

Camillo Costantini

Dipartimento di Matematica, Università di Torino, Italy

We consider the problem of partitioning the real plane — or suitable subsets of it — into (non-degenerated) closed segments. In particular, we show that:

- (1) *It is possible to find a partition of \mathbb{R}^2 into closed segments, all having the same length.*
- (2) *It is possible to find a partition of \mathbb{R}^2 into closed segments, each of which is parallel to either the first or the second co-ordinate axis. Moreover, we may require each element of such a partition to have a length chosen among two pre-assigned ones.*

Let also call a subset of the real plane *segment-free*, if it contains no non-degenerated closed segment. We prove that there is a subset of the real plane, with segment-free complement, which may be partitioned into countably many closed segments. The construction is based on the idea of creating inductively an infinite grill, by adding at each stage segments of decreasing length, which are alternately parallel to the first or the second co-ordinate axis.

References

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