ITES2001 Fourth Italian-Spanish conference on General Topology and its applications

Bressanone, 27-30 June 2001

On a game in a topological group

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Let G be a topological group. The game $\mathfrak{G}(G)$ has two players $P1$ and $P2$. Player P1 chooses an element $x \in G$ and player P2 tries to guess what element x has been chosen by $P1$ by asking questions of the following form. For a sequences $\underline{m} = \{m_n\}$ of integers the question $Q_{\underline{m}}$ is: does x^{m_n} converge to 1 in G when $n \to \infty$? The aim of the player P2 is to accumulate as much information about x as possible knowing the answers of all $\mathfrak{c}$ many questions $Q_{\underline{m}}$, i.e., the set $S(x) = \{\{m_n\} \in \mathbb{Z}^\omega : \lim x^{m_n} = 1\}$. Since $S(x) = S(x^{-1})$, $P2$ can guess at most the *doubleton* $\{x, x^{-1}\}$, in case x is non-torsion, or the set $\mathrm{gen}(x)$ of all generators of the cyclic group $\langle x \rangle$. In such a case we say that $P2$ wins (in other words, P2 wins if $S(x) = S(y)$ always implies $\langle x \rangle = \langle y \rangle$ in G). We prove that:

(a) P2 wins the game $\mathfrak{G}(G)$ for a non-discrete locally compact group G iff G is isomorphic to the circle group;

(b) P2 wins the game $\mathfrak{G}(G)$ for a discrete group G iff G is isomorphic to a subgroup of the dicrete group $\mathbb{Q}/\mathbb{Z}$.

We also discuss the groups G where the game $\mathfrak{G}(G)$ is most optimal for player P1, namely those with minimum information for P2 (i.e., $S(x) = S(y)$ whenever x, y are non-trivial elements of G, e.g., the reals $\mathbb{R}$, the group of p-adic numbers, etc.). We give a complete characterization of these groups G in the class of locally compact abelian groups. The proofs are based on the structure theory of locally compact groups and properties of topologically torsion elements (cf. [1]).

References

[1] D. Dikranjan, *Answer to a question of Armacost on topologically torsion elements*, Second Honolulu Conference on Abelian Groups and Modules, July 25 – August 1, 2001, Hawaii, Preprint.