

The existence of dense pseudocompact subgroups and of pseudocompact refinements

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The 1966 results of Comfort and Ross revealed pseudocompact groups as one of those objects for which the confluence of algebra and topology yields a richer structure. Pseudocompact groups have since drawn the attention of many general topologists, who find among them a setting amenable to their techniques.

The topic of this talk belongs to this tradition and will be centered on the following two extremal phenomena involving a pseudocompact group (G, \mathcal{T}) :

- (1) Can (G, \mathcal{T}) be extremal in the sense that no finer group topology $\mathcal{S} \supset \mathcal{T}$ on G is pseudocompact?
- (2) Can (G, \mathcal{T}) be extremal in the sense that it contains no proper dense subgroup which is pseudocompact?

Research around problems of this sort began with a paper by Comfort and Soundararajan [2], and took its present form with the monograph [1] where, among other results, it was proved that compact Abelian groups are not extremal in either of the senses 1 or 2 above.

Yet the question about the existence of (any kind of) extremal pseudocompact groups remains unsolved, a number of particular cases show that extremality is rather a pathological property and support the conjecture (with no evidence, though) that no pseudocompact Abelian group is extremal in either sense.

The style of the talk will be chiefly expository. We shall attempt to outline the main techniques employed to find finer pseudocompact topologies and dense pseudocompact subgroups, to describe the deep ties existing between the two extremal properties, and to show which Abelian pseudocompact groups are the (few) remaining candidates for extremality. Some unpublished results obtained jointly with W. W. Comfort will be included.

References

- [1] W. W. Comfort and Lewis C. Robertson. Extremal phenomena in certain classes of totally bounded groups. *Dissertationes Math. (Rozprawy Mat.)*, 272, 1988.
- [2] W. W. Comfort and T. Soundararajan. Pseudocompact group topologies and totally dense subgroups. *Pacific J. Math.*, 100(1), 61–84, 1982.