ITES2001 Fourth Italian-Spanish conference on General Topology and its applications

Bressanone, 27-30 June 2001

On the Poincaré-Bendixson Theorem

Gabriel Soler López
Universidad de Cartagena, Spain

The celebrated Poincaré-Bendixson Theorem assures that given a C^1-flow on $\mathbb{S}^2$, $\Psi : \mathbb{R} \times \mathbb{S}^2 \to \mathbb{S}^2$, and a point x whose ω-limit set does not contains any singular point, then $\omega_\Phi(x)$ is a closed orbit. The aim of this talk is to show in which phase spaces this Theorem also works, in particular we will see that it is valid for the Klein Bottle and the Projective Plane with the same formulation that in the case of $\mathbb{S}^2$.

In all the compact and connected surfaces, except for the Torus, we will see that the Theorem works for flows of C^2-class. Finally we will state the recent results in the Poincaré-Bendixson Theory, namely we will state topological characterizations of the ω-limit sets in some phase spaces: Klein Bottle, Projective Plane and n-dimensional sphere.