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Products in the strong shape category

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Direct products are defined in arbitrary categories and are unique, whenever they exist. In the category H(Top) of topological spaces and homotopy classes of mappings, for any two spaces X, Y their product exists and is given by the Cartesian product $X \times Y$ and by the homotopy classes $[\pi_X]$ and $[\pi_Y]$ of the canonical projections $\pi_X : X \times Y \to X$, $\pi_Y : X \times Y \to Y$, respectively. Since shape theory is a modification of homotopy theory, it is natural to ask whether in the shape category Sh(Top) the Cartesian product $X \times Y$ and the shape morphisms $S[\pi_X], S[\pi_y]$, induced by $[\pi_X]$ and $[\pi_Y]$, form the direct product of X and Y ? In 1974 J.E. Keesling [1] exhibited a simple (non-compact) space $X \subseteq \mathbb{R}^2$ such that $X \times X$ and the two shape morphisms $S[\pi_X] : X \times X \to X$ do not form a product in Sh(Top). However, he gave a positive answer in the case when X and Y are compact Hausdorff spaces. In 1977 Y. Kodama proved that the answer is positive also in the case when X is an FANR (fundamental absolute neighorhood retract) and Y is a paracompact space [2].

The main aim of the present talk is to announce analogous positive results in strong shape [4], i.e. for the strong shape category SSh(Top) and the strong shape functor $\overline{S} : \mathrm{H(Top)} \to \mathrm{SSh(Top)}$ (for definitions see [3]).

THEOREM 1. *If X and Y are compact Hausdorff spaces, then $X \times Y$, $\overline{S}[\pi_X]$ and $\overline{S}[\pi_Y]$ form the product of X and Y in* SSh(Top).

THEOREM 2. *If X is an* FANR *and Y is a finite-dimensional space* (*more general, a finitistic space*), *then $X \times Y$, $\overline{S}[\pi_X]$ and $\overline{S}[\pi_Y]$ form the product of X and Y in* SSh(Top).

References

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